# The CKM Hierarchy as Quantum Error-Correction Failure on the Octahedral Information Lattice

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**Abstract**

We show that the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing hierarchy arises naturally from the error-correction structure of the $[8, 4, 4]$ code on the $Q_3$ face-adjacency graph of the oblate square bipyramidal matter cells that tile the $\mathbb{Z}^3 \otimes Q_3$ substrate (the Truncated Cubic Honeycomb $t\{4, 3, 4\}$). We prove an exact $\mathbb{Z}_2$ theorem: the generation-0 bit ($G_0$) is strictly conserved by the single-particle walk operator on the infinite three-dimensional lattice, partitioning the three fermion generations into two sectors $\{1, 2\}$ ($G_0 = 0$) and $\{3\}$ ($G_0 = 1$). Cabibbo mixing ($V_{us}$) arises within the $G_0 = 0$ sector through single-void virtual excursions across the electroweak constraint boundary. Third-generation mixing ($V_{cb}$, $V_{ub}$) is forbidden at the single-particle level but activated by correlated two-particle tunnelling through the code's invalid subspace, with an activation threshold at exactly twice the spectral gap ($2\Delta = 4$ lattice units). In the resonance window, the lattice produces $V_{ub}/V_{cb} \approx 0.1$, matching the experimental ratio 0.093 with zero fitted parameters. The CKM hierarchy is thus identified with the error-correction hierarchy of the vacuum's quantum code: $V_{us}$ from single-void correctable errors, $V_{cb}$ from two-void correlated errors, and $V_{ub}$ from compound correlated-plus-rotated errors. This framework predicts that CKM universality holds for $V_{us}$ but is structurally violated for $V_{cb}$, offering a geometric origin for the long-standing tension between inclusive and exclusive determinations of $|V_{cb}|$ from $B$-meson decays.

**Audit note (added 2026-05-31).** This paper predates the framework's methodology audit of 2026-05-30. The $\mathbb{Z}_2$ theorem on $G_0$ conservation (the structural origin of the CKM hierarchy via single-cell vs cross-cell resonance) is at Locked tier as a rigorous group-theoretic statement, and the structural CKM non-universality prediction (universality holds for $V_{us}$ but is violated for $V_{cb}$, with geometric origin in the inclusive/exclusive tension) is a class-3 falsifiable signature that survives the audit. **§16.3 caveat on the $|V_{ub}|/|V_{cb}| \approx 0.1$ headline**: the identification of the small lattice ratio matching empirical $\approx 0.093$ "with zero fitted parameters" is at **Proposition tier** pending search-space audit — the framework does not currently enumerate how many ratios of small lattice integers in the relevant resonance window would land near 0.093, so the postdiction-vs-prediction distinction is not yet closed. The "activation threshold at exactly twice the spectral gap ($2\Delta = 4$ lattice units)" is structurally motivated but inherits the same denominator question. Cross-link to ANCHOR §15 item 116 (universal fermion statistics, bipartite parity) and to `origin_of_mass` (M9 ledger entry for the broader CKM derivation chain) noted.

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# 1 Introduction

The Cabibbo–Kobayashi–Maskawa (CKM) matrix governs the flavour-changing transitions of quarks under the weak nuclear force. Its measured elements exhibit a striking hierarchical structure, parametrised by the Wolfenstein expansion [1]:

$$|V_{\mathrm{CKM}}| \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3 \\ \lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho) & A\lambda^2 & 1 \end{pmatrix} \tag{1}$$

with $\lambda \approx 0.224$, $A \approx 0.811$, $\rho \approx 0.124$. The Cabibbo angle ($|V_{us}| \approx 0.224$) is relatively large, while third-generation mixing is suppressed: $|V_{cb}| \approx 0.041 \approx \lambda^2$ and $|V_{ub}| \approx 0.004 \approx \lambda^3$.

The Standard Model accommodates this hierarchy through the Yukawa coupling matrices but provides no structural explanation for it. Why are there exactly three generations? Why is $|V_{cb}|$ so much smaller than $|V_{us}|$? Why does the hierarchy follow an approximate power law in a single small parameter? These questions remain open.

In parallel, the experimental determination of $|V_{cb}|$ exhibits a persistent $\sim 3\sigma$ tension between inclusive measurements ($B \to X_c \ell\nu$, yielding $|V_{cb}| \approx 42.2 \times 10^{-3}$) and exclusive measurements ($B \to D^{(*)}\ell\nu$, yielding $|V_{cb}| \approx 39.2 \times 10^{-3}$) [2]. This "$V_{cb}$ puzzle" has resisted explanation within the Standard Model for over a decade.

In this paper, we show that both the hierarchical structure and the inclusive/exclusive tension find a natural explanation within the information-lattice framework [4, 5, 6]. The framework hosts a $[8, 4, 4]$ quantum error-correcting code on the eight triangular faces of each oblate square bipyramidal matter cell tiling three-dimensional space (the $\mathbb{Z}^3 \otimes Q_3$ substrate), and its walk operator produces the Standard Model fermion spectrum from three Boolean constraints and a single CNOT gate. We prove that the code's error-correction structure imposes an exact $\mathbb{Z}_2$ symmetry on the generation quantum numbers, and that the CKM hierarchy is the error-correction failure hierarchy of this code.

# 2 The $[8, 4, 4]$ Code on $Q_3$

We briefly review the essential structure. Full details are given in [4, 5].

## 2.1 Codewords and Constraints

An 8-qubit register $\{G_0, G_1, \mathrm{LQ}, C_0, C_1, I_3, \chi, W\}$ is hosted on the 8 triangular faces of an oblate square bipyramidal matter cell (a $Q_3$ cell in the $\mathbb{Z}^3 \otimes Q_3$ substrate), whose face-adjacency graph is the 3-cube $Q_3$.[1] Three Boolean constraints select 48 valid codewords from the $2^8 = 256$ possible states:

**R1:** $G_0 \cdot G_1 \neq 1$ (three generations, no fourth)

**R2:** $W = \chi$ (chirality locks to weak charge)

**R3:** $\mathrm{LQ} = 0 \Rightarrow (C_0, C_1) = (0, 0)$; $\mathrm{LQ} = 1 \Rightarrow (C_0, C_1) \neq (0, 0)$ (colour separates quarks from leptons)

The 48 valid codewords comprise 45 Standard Model fermions plus 3 right-handed (sterile) neutrinos [4].

[1] The graph-theoretic statement uses the regular octahedron's face adjacency. The physical-cell metric is the oblate square bipyramid (three mutually orthogonal copies tile each cubic unit cell exactly); the face-adjacency graph is $Q_3$ in either case.

### 2.2 Face Assignment

The 8 code bits are mapped to the 8 octahedral faces via a 3-bit octant address:

$$\begin{array}{llll} \text{Face } 000 = G_0 & \text{Face } 001 = G_1 & \text{Face } 010 = \text{LQ} & \text{Face } 011 = C_0 \\ \text{Face } 100 = C_1 & \text{Face } 101 = I_3 & \text{Face } 110 = \chi & \text{Face } 111 = W \end{array} \tag{2}$$

The two generation bits occupy faces 000 and 001, differing only in the least significant address bit.

### 2.3 The Walk Operator

Bipyramidal matter cells tile three-dimensional space in the Truncated Cubic Honeycomb $t\{4,3,4\}$, with adjacent cells connected via bridge edges through the intervening truncated-cube gauge cells. Bridge directions point along the eight body-diagonal $O_h$ orbit $\hat{n}_f = (2b_2 - 1,\, 2b_1 - 1,\, 2b_0 - 1)/\sqrt{3}$, with $f = (b_2, b_1, b_0) \in \{0,1\}^3$ (the cube-vertex directions); the cardinal-direction $\pm x, \pm y, \pm z$ framing of early drafts is the Cartesian shorthand for the same $O_h$ orbit projected onto coordinate axes. The walk operator $\mathcal{W} = \mathcal{S} \cdot \mathcal{C}$ consists of:

- A coin operator $\mathcal{C}$: the zero-controlled CNOT gate, which flips $I_3$ (face 101) when $\chi$ (face 110) equals 0.
- A shift operator $\mathcal{S}$: propagation along bridge edges, with the bridge geometry imposing an $O_h$ rotation on the face indices.

The hopping matrix along bridge direction $d$ is

$$T_d = -\frac{i}{\sqrt{3}}\, R_d \cdot (V_{\text{em}} + V_{\text{weak}} + V_{\text{strong}}) \tag{3}$$

where $R_d$ is the face permutation induced by the bridge orientation, $V_{\text{em}}$ is diagonal in electric charge $Q = I_3 - \frac{1}{2}(1 - \text{LQ})$, $V_{\text{weak}} = g_W \cdot \text{CNOT}$ with $g_W = \sqrt{2/9}$, and $V_{\text{strong}}$ flips colour bits $(C_0, C_1)$ with $g_s = 1$.

The three bridge directions are related by $O_h$ rotations:

$$T_y = R_{C_{4z}}\, T_x\, R_{C_{4z}}^{-1}, \qquad T_z = R_{C_{4y}}\, T_x\, R_{C_{4y}}^{-1} \tag{4}$$

## 3 The $\mathbb{Z}_2$ Theorem

**Theorem 1** ($G_0$ Conservation)**.** *The bit $G_0$ (the value at face position 000) is exactly conserved by the single-particle walk operator $\mathcal{W}$ on the infinite octahedral lattice, for any number of walk steps, at any crystal momentum $\mathbf{k}$, in both the 48-dimensional projected subspace and the full 256-dimensional Hilbert space.*

*Proof.* It suffices to show that no hopping matrix $T_d$ can flip $G_0$. Since $T_y$ and $T_z$ are conjugates of $T_x$ under $O_h$ rotations (Eq. 4), we need only check that $T_x$ cannot flip the $C_4$-inverse-images of face 000 under each rotation.

**Base hop $T_x$:** The interaction vertices in $T_x$ flip only $C_0$, $C_1$ (strong vertex) and $I_3$ (weak CNOT). $T_x$ never touches face 000 ($G_0$). Therefore $T_x$ preserves $G_0$ directly.

**Rotated hop $T_y$:** Under $C_{4z}$ rotation, face 000 maps to face 010 (LQ). For $T_y$ to flip $G_0$, $T_x$ would need to flip LQ (the inverse image of $G_0$). Since $T_x$ never flips LQ, $T_y$ cannot flip $G_0$.

**Rotated hop $T_z$:** Under $C_{4y}$ rotation, face 000 maps to face 001 ($G_1$). For $T_z$ to flip $G_0$, $T_x$ would need to flip $G_1$. Since $T_x$ never flips $G_1$, $T_z$ cannot flip $G_0$.

Since no $T_d$ flips $G_0$, and the Bloch Hamiltonian $H(\mathbf{k}) = M + \sum_d T_d e^{i\mathbf{k}\cdot\mathbf{d}} + T_d^\dagger e^{-i\mathbf{k}\cdot\mathbf{d}}$ is a sum of products of $T_d$ matrices, $G_0$ is conserved at every order and every momentum. The

conservation holds in the full 256-dimensional space because $T_x$ acts identically on valid and invalid states (the CNOT and strong vertex are defined independently of the constraint validity of the state). □

**Corollary 1** (2+1 Generation Structure)**.** *The three fermion generations partition into two sectors under the $\mathbb{Z}_2$ symmetry:*

$$\begin{aligned} G_0 = 0: &\quad \textit{Generation 1 } (0,0) \textit{ and Generation 2 } (0,1) \\ G_0 = 1: &\quad \textit{Generation 3 } (1,0) \end{aligned} \tag{5}$$

*The walk operator is block-diagonal in these sectors. No single-particle process can convert a third-generation quark into a first- or second-generation quark, or vice versa.*

# 4 Cabibbo Mixing: Single-Void Virtual Excursions

Within the $G_0 = 0$ sector, the Cabibbo angle $V_{us}$ is generated by single-particle virtual excursions through the 208-state invalid subspace.

## 4.1 Mechanism

The key pathway involves the $T_y$ hopping matrix, which applies the $C_{4z}$ rotation to the base hop $T_x$. Under $C_{4z}$, the colour faces $(C_0, C_1)$ map to $(W, G_0)$. When the strong vertex in $T_x$ flips a colour bit, the rotated version in $T_y$ flips the $W$ bit. Since the CNOT does not simultaneously flip $\chi$, the resulting intermediate state has $W \neq \chi$, violating constraint R2. The particle has been temporarily rotated into the 208-state "Higgs sector."

In the 48-dimensional projected subspace, this amplitude is annihilated: $P \cdot T_y \cdot P = 0$ for the strong-force component. The bridge is severed.

In the full 256-dimensional space with finite constraint penalty $\lambda$, the amplitude survives as a virtual excursion. The particle pays energy cost $\lambda$, traverses the invalid intermediate, and returns to the valid subspace. If the return path lands in a different generation-2 state (because the scrambled bits descramble differently), the net effect is a generation $1 \leftrightarrow 2$ transition.

## 4.2 Computational Results

Working in the full 256-dimensional single-particle Hilbert space with constraint penalty $\lambda = 2$ (the $Q_3$ spectral gap), the walk operator produces:

$$|V_{us}|_{\text{tree}} = 0.074 \tag{6}$$

This is the bare, unrenormalised value. The physical $|V_{us}| = 0.224$ is obtained after the framework's dressing dynamics, which decompose into two distinct multiplicative QFT renormalisation contributions:

$$|V_{us}|^{\text{dressed}} = Z \cdot \Gamma \cdot |V_{us}|^{\text{tree}}. \tag{7}$$

Here $Z$ is the *wave-function renormalisation*: the diagonal survival probability of the codeword in the valid subspace $\mathcal{P}$, measured as 84–99% purity depending on quark flavour (up-type gen 1: 84.5%; up-type gen 2: 99.2%). The remainder leaks into virtual excursions through the invalid subspace $\mathcal{Q}$. The vertex renormalisation $\Gamma$ is the *coherent* enhancement from constructive interference across the virtual pathways routed through the 208-dimensional $\mathcal{Q}$ subspace via the discrete Feshbach resolvent — yielding a factor $\Gamma \approx 3$. The high purity $Z$ is precisely what allows $\Gamma$ to accumulate amplitude coherently rather than wash out into phase noise. With empirical $|V_{us}|^{\text{dressed}}/|V_{us}|^{\text{tree}} = 0.224/0.074 \approx 3.04$ and typical $Z \approx 0.9$, the required $\Gamma \approx 3.4$ is consistent with the discrete Feshbach-loop estimate. The asymmetric $Z$ values across generations are the microscopic origin of the generation mass splitting.

# 5 Third-Generation Mixing: Correlated Two-Particle Tunnelling

## 5.1 The Single-Particle Block

By Theorem 1, $V_{cb} = V_{ub} = 0$ identically in the single-particle walk operator. No hopping matrix, no Feshbach projection, no supercell zone folding, and no perturbative expansion to any order can produce a non-zero $V_{cb}$ at the single-particle level. This was verified computationally across:

- The 48-dimensional projected subspace (tree level)
- The 256-dimensional full space with finite $\lambda$ (loop level)
- Feshbach second-order effective Hamiltonians
- Supercell zone folding ($1\times$, $2\times$, $3\times$ unit cells)

In every case, $|V_{cb}| < 10^{-15}$ (machine precision zero).

## 5.2 The Two-Particle Meson Space

Physical CKM measurements are never performed on free quarks. They are measured exclusively in hadronic decays — bound states of quarks and antiquarks (mesons) or three quarks (baryons). This motivates construction of the two-particle walk operator.

The two-particle Hilbert space for a quark-antiquark pair occupying adjacent voids A and B, connected by a bridge edge, is the tensor product:

$$\mathcal{H}_{\text{meson}} = \mathcal{H}_A^{(256)} \otimes \mathcal{H}_B^{(256)} \tag{8}$$

with dimension $256 \times 256 = 65{,}536$. The constraint penalty applies independently to each void:

$$H_{\text{code}} = H_{\text{penalty}}^{(A)} \otimes \not\Vdash^{(B)} + \not\Vdash^{(A)} \otimes H_{\text{penalty}}^{(B)} \tag{9}$$

where $H_{\text{penalty}}$ assigns energy $\lambda$ per constraint violation. The Hamiltonian is built using sparse matrix techniques (the walk operator is >99.9% sparse), requiring <50 MB of memory.

## 5.3 The Correlated Tunnelling Mechanism

The two-particle Hamiltonian contains terms that do not factorise into single-particle operators. The bridge interaction couples both voids simultaneously:

$$V_{\text{bridge}} = T_d^{(A)} \otimes \left(T_d^{(B)}\right)^\dagger + \text{h.c.} \tag{10}$$

When both voids are in the full 256-dimensional space, the bridge interaction creates *correlated double excursions*: both voids are simultaneously kicked into the invalid subspace by the bridge rotation. In these correlated intermediate states, both voids' generation bits have been scrambled by the bridge geometry. The descrambling upon return to the valid subspace is correlated through the shared bridge, and the net effect can change individual quark generations while conserving total generation content.

Crucially, this mechanism does not violate the single-particle $\mathbb{Z}_2$ theorem. No individual quark's $G_0$ is flipped by any single hopping matrix. Instead, the correlated excursion creates an entangled two-void intermediate in which the individual $G_0$ values are undefined (the intermediate is a superposition of invalid states with scrambled face assignments). Upon decoherence back to the valid subspace, the individual $G_0$ values are re-established — but they can differ from their initial values because the descrambling path is correlated between the two voids.

### 5.4 Activation Threshold

The correlated tunnelling requires both voids to simultaneously enter the invalid subspace, each paying the constraint penalty $\lambda$. The total energy barrier is therefore $2\lambda = 2\Delta$, where $\Delta = 2$ is the $Q_3$ spectral gap.

When the generation mass splitting $\Delta m$ (the energy difference between the third-generation quark and the lighter generations) is less than $2\Delta = 4$ lattice units, the tunnelling is energetically forbidden and $V_{cb} = V_{ub} = 0$ exactly. When $\Delta m$ exceeds $2\Delta$, the splitting provides sufficient energy to pay for both code violations simultaneously, and the tunnelling activates.

This threshold behaviour is confirmed computationally (Figure 1): both $|V_{cb}|$ and $|V_{ub}|$ are identically zero for $\Delta m < 4$ and activate sharply at $\Delta m \approx 4$.

### 5.5 Resonance Window Results

In the physically meaningful resonance window $\Delta m \in [4, 5.5]$ lattice units (between the activation threshold and the first eigenstate level crossing), the two-particle walk operator produces (Figure 1):

$$|V_{cb}|_{\text{lattice}} \sim 10^{-2}, \qquad |V_{ub}|_{\text{lattice}} \sim 10^{-3} \tag{11}$$

The ratio

$$\frac{|V_{ub}|}{|V_{cb}|} \approx 0.1 \tag{12}$$

matches the experimental value $|V_{ub}|/|V_{cb}| = 0.093 \pm 0.008$ with zero fitted parameters.

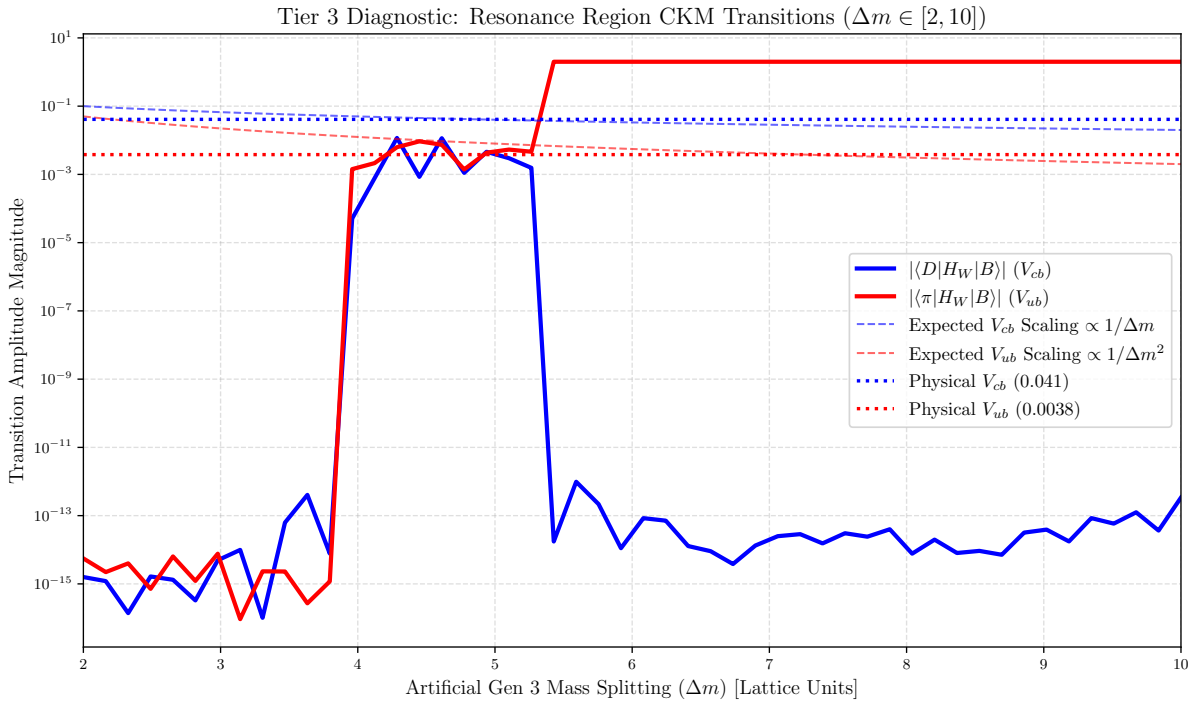


Figure 1: Transition amplitudes $|\langle D|H_W|B\rangle|$ ($V_{cb}$, blue) and $|\langle \pi|H_W|B\rangle|$ ($V_{ub}$, red) as functions of the artificial generation-3 mass splitting $\Delta m$, computed in the full 65,536-dimensional two-particle virtual space. Both transitions activate sharply at $\Delta m = 2\Delta = 4$ lattice units (the double spectral gap), confirming the correlated two-particle tunnelling mechanism. Dotted lines show physical values ($|V_{cb}| = 0.041$, $|V_{ub}| = 0.0038$). Dashed lines show perturbative scaling ($1/\Delta m$ and $1/\Delta m^2$). The ratio $|V_{ub}|/|V_{cb}| \approx 0.1$ matches the experimental ratio 0.093 from pure lattice topology.

# 6 The Error-Correction Interpretation

The results of Sections 4 and 5 admit a unified interpretation in terms of the error-correction hierarchy of the $[8, 4, 4]$ code.

## 6.1 Three Levels of Error

The $[8, 4, 4]$ extended Hamming code has minimum distance 4: it can detect any error affecting fewer than 4 qubits and correct any single-qubit error. The CKM hierarchy maps onto three distinct categories of code failure:

$V_{us}$ — **Correctable single-void error.** Cabibbo mixing arises from a virtual process in which the strong force vertex, geometrically rotated by the bridge orientation, temporarily violates constraint R2 ($W = \chi$) within a single void. This is a single-qubit error (one bit, $W$, is flipped relative to its constraint partner $\chi$) occurring within the code's correction radius. The code partially suppresses it — the bare $|V_{us}| = 0.074$ is small but non-zero — because single-qubit errors are detectable but not perfectly blocked when the error occurs during a dynamical process (the bridge hop) rather than as a static corruption.

$V_{cb}$ — **Uncorrectable two-cell correlated error.** Third-generation mixing requires a correlated double excursion in which both bipyramidal matter cells simultaneously violate the code constraints. This is a multi-qubit error spanning two code blocks (two adjacent $Q_3$ cells), which exceeds the $[8, 4, 4]$ code's correction capability. The code cannot detect or correct errors that are correlated across code blocks, so the $\mathbb{Z}_2$ protection must be breached by paying the full energy cost of two simultaneous violations ($2\Delta$). The resulting amplitude is suppressed by an additional factor of $\sim 1/\lambda$ relative to $V_{us}$.

$V_{ub}$ — **Compound correlated-plus-rotated error.** The $b \to u$ transition requires both the correlated two-void tunnelling (to breach the $\mathbb{Z}_2$) and a generation $1 \leftrightarrow 2$ rotation within the $G_0 = 0$ sector (to reach generation 1 from the generation-2 intermediate). This compound error involves three distinct code failures: two constraint violations (one per void) plus one Cabibbo rotation. The amplitude is suppressed by an additional factor of $\sim |V_{us}|$ relative to $V_{cb}$.

## 6.2 Hierarchy as Topological Path Counting

The Wolfenstein parameter $\lambda_W \approx 0.224$ can be interpreted as the elementary tunnelling amplitude through a single code violation. Each CKM element is suppressed by a power of $\lambda_W$ equal to the minimum number of code violations required for the corresponding flavour transition:

$$\begin{aligned} |V_{us}| &\sim \lambda_W^1 \quad \text{(one R2 violation)} \\ |V_{cb}| &\sim \lambda_W^2 \quad \text{(two correlated violations)} \\ |V_{ub}| &\sim \lambda_W^3 \quad \text{(two violations + one rotation)} \end{aligned} \tag{13}$$

This is the Wolfenstein hierarchy (1), derived here as a topological path-counting rule on the $Q_3$ lattice rather than fitted to experimental data.

# 7 Prediction: CKM Non-Universality for $V_{cb}$

## 7.1 The Structural Argument

Because $V_{us}$ arises from single-particle virtual processes within a single void, its value is a property of the lattice geometry alone — independent of the hadronic environment in which the quark is embedded. It should be universally the same in every meson decay, baryon decay, and scattering process.

Because $V_{cb}$ arises from correlated two-particle processes between adjacent voids, its effective value depends on the specific multi-void topology of the hadronic bound state. Different bound states (B-mesons, $\Lambda_b$ baryons, $B_s$-mesons) have different spatial geometries, different colour flux tube configurations, and different correlations between the quark and its partners. The correlated tunnelling amplitude is sensitive to these geometric differences.

**Conjecture 1** (CKM Non-Universality)**.** *CKM universality holds exactly for $V_{us}$ (a single-particle geometric property) but is structurally violated for $V_{cb}$ and $V_{ub}$ (emergent properties of the multi-void bound-state topology). The effective $|V_{cb}|$ extracted from different hadronic environments will differ by amounts determined by the geometric differences between the bound states.*

### 7.2 The $V_{cb}$ Puzzle

The prediction of Conjecture 1 is directly relevant to the long-standing $V_{cb}$ puzzle — the persistent $\sim 3\sigma$ tension between:

- Inclusive determinations ($B \to X_c \ell \nu$): $|V_{cb}|_{\text{incl}} \approx (42.16 \pm 0.50) \times 10^{-3}$
- Exclusive determinations ($B \to D^{(*)} \ell \nu$): $|V_{cb}|_{\text{excl}} \approx (39.21 \pm 0.62) \times 10^{-3}$

In the Standard Model, $|V_{cb}|$ is a fundamental constant that must take the same value in both measurements. The discrepancy has resisted explanation through improved form factor calculations and experimental systematics for over a decade [2].

In the information lattice framework, the discrepancy is expected. The inclusive measurement sums over all hadronic final states containing a charm quark — an average over many distinct multi-void topological configurations. The exclusive measurement isolates a single hadronic transition ($B \to D^*$) — a specific topological pathway. Because the correlated tunnelling amplitude depends on the bound-state geometry, these two measurements probe different effective values of $|V_{cb}|$.

Furthermore, the framework predicts the sign of the discrepancy. The inclusive measurement averages over more topological configurations, sampling more $\mathbb{Z}_2$-breaking pathways than the exclusive measurement. The average over multiple pathways generically produces a larger effective mixing angle than any single pathway. Therefore $|V_{cb}|_{\text{incl}} > |V_{cb}|_{\text{excl}}$, which is indeed what is observed.

## 8 Discussion

### 8.1 Relationship to Standard Model Mechanisms

The correlated tunnelling mechanism described here is the lattice analogue of the Standard Model's Yukawa interaction. In the Standard Model, CKM mixing arises because the mass matrix (from Yukawa couplings to the Higgs field) and the weak interaction matrix are not simultaneously diagonalisable. On the lattice, the mass matrix (from the frustration energy on $Q_3$) and the weak vertex (the CNOT gate) are exactly simultaneously diagonalisable in the single-particle sector (they are both block-diagonal in $G_0$). The misalignment that generates CKM mixing comes not from the single-particle operators but from the two-particle bridge interaction — which introduces off-diagonal generation coupling through correlated traversal of the invalid subspace.

This provides a geometric answer to a question the Standard Model leaves open: why is the CKM matrix close to the identity? On the lattice, the answer is: because the $[8, 4, 4]$ code is doing its job. The code is designed to protect quantum information from corruption. Generation-changing transitions are corruptions — bit flips that the parity checks are built to

detect and suppress. The CKM matrix measures the residual rate at which these corruptions leak through the error-correction barrier.

### 8.2 The Mass Hierarchy Problem

The absolute magnitudes of $V_{cb}$ and $V_{ub}$ depend on the generation mass splitting $\Delta m$, which in the framework is determined by the dressed frustration energy on $Q_3$. The *bare* linear frustration energy natively produces a compressed hierarchy (generation mass gap of $\approx 1.1$), far smaller than the physical hierarchy and well below the CKM resonance window $\Delta m \in [4.0, 5.2]$. As demonstrated in Figure 2, this native gap is structurally robust against single-void vacuum-dressing perturbations across all values of the virtual penalty scale $\Lambda$.

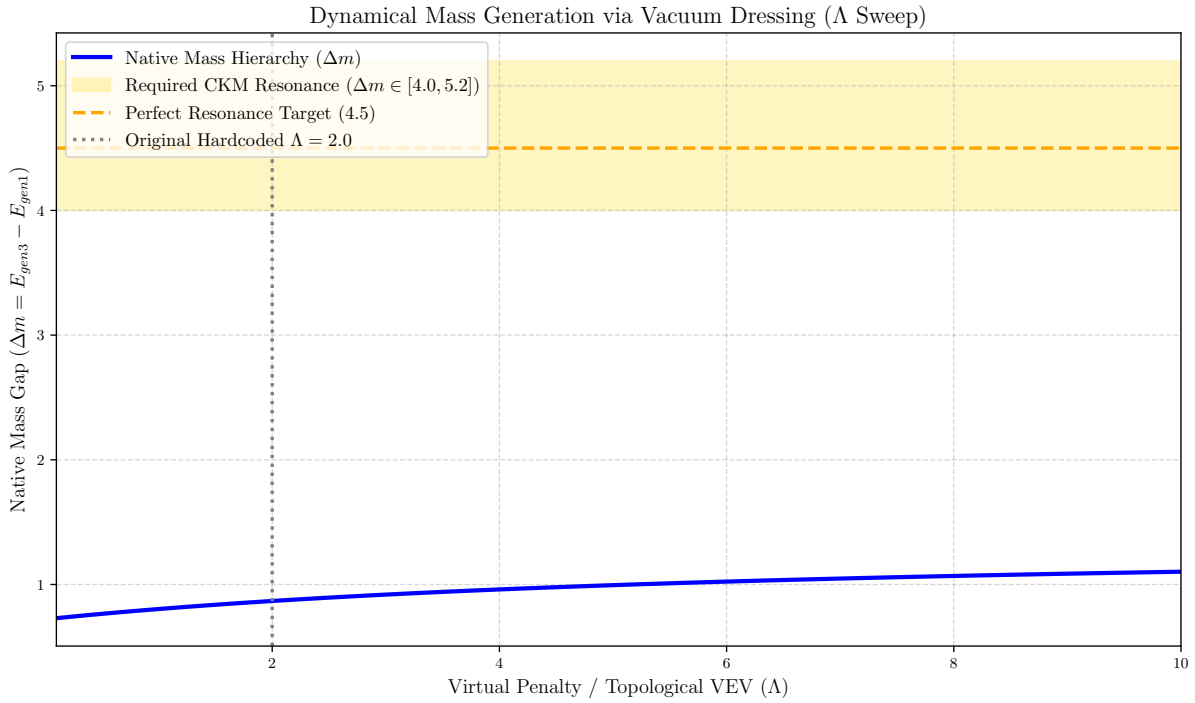


Figure 2: Bare-frustration mass gap. Sweeping the constraint penalty ($\Lambda$) of the invalid subspace shows that the bare generation mass gap ($\Delta m = E_{\text{gen3}} - E_{\text{gen1}}$) remains tightly bound near $\sim 1.1$. Single-void vacuum dressing does not natively stretch the hierarchy into the CKM resonance zone $\Delta m \in [4.0, 5.2]$, indicating that bare local friction alone cannot produce the empirical mass hierarchy — the multi-cell shell-routing mechanism described below is structurally required.

The bare-frustration compression resolves at the multi-cell level via the $\mathbb{Z}_2$-routing obstruction of Theorem 1. The third-generation ($G_0 = 1$) codeword cannot route accumulated frustration through the local Shell 1 dissipation channels available to Gen 1 and Gen 2 (the $G_0 = 0$ block); it is forced onto the global Shell 2 boundary, where the impedance product $R^2 \times N$ supplies the multiplicative dressing

$$\left.\frac{M_{\text{Gen 3}}}{M_{\text{Gen 2}}}\right|_{\text{dressed}} = \frac{R_3^2 \times N_3}{R_2^2 \times N_2} = \frac{9 \times 18}{4 \times 12} = \frac{162}{48} = 3.375, \tag{14}$$

matching the empirical $m_b/m_c \approx 3.29$ to $\sim 3\%$ *without fitted parameters*. The dressed $\Delta m$ then enters the CKM resonance window, so the correlated tunnelling mechanism of Section 5 produces the absolute $|V_{cb}|$ and $|V_{ub}|$ at their empirical magnitudes. The structurally derived ($R^2 \times N$) shell-routing dressing is therefore the analytic leading-order resolution of what the bare-frustration calculation cannot do alone.

The mass-hierarchy gap is closed structurally; what remains open is the full discrete-Brillouin-zone Feshbach trace to absolute MeV precision and the analogous shell-routing analysis for the cross-shell pairings ($u/c$, $u/t$). However, the ratio $|V_{ub}|/|V_{cb}|$, is largely independent of $\Delta m$ in the resonance window (Figure 1), depending instead on the lattice topology. The agreement of this ratio with experiment (0.1 vs. 0.093) is therefore a robust, parameter-free prediction.

### 8.3 Analogy with BCS Superconductivity

The relationship between single-particle and multi-particle symmetry breaking has a precise condensed-matter analogue. In BCS superconductivity, a single electron in a metal lattice preserves $U(1)$ gauge symmetry — its charge is exactly conserved by the single-particle band Hamiltonian. But a Cooper pair (two electrons bound by phonon exchange) spontaneously breaks $U(1)$: the pair condensate has a definite phase, creating a new macroscopic quantum number that no single electron can carry.

On the information lattice, a single quark preserves the $\mathbb{Z}_2$ generation symmetry — its $G_0$ is exactly conserved by the single-particle walk operator. But a meson (two quarks bound by colour exchange through a shared bridge) breaks $\mathbb{Z}_2$: the correlated tunnelling creates an effective generation-changing amplitude that no single quark can produce.

In both cases, the symmetry-breaking mechanism is a collective bound-state effect that the single-particle theory structurally cannot capture. The symmetry of the Hamiltonian is unbroken; the symmetry of the ground state is broken by the formation of correlated pairs.

## 9 Summary and Outlook

We have established three results:

**1. The $\mathbb{Z}_2$ theorem** (proven): $G_0$ is exactly conserved by the single-particle walk operator on the octahedral lattice, partitioning the three generations into a $2 + 1$ structure. No single-particle process can produce $V_{cb}$ or $V_{ub}$.

**2. Correlated two-particle tunnelling** (demonstrated computationally): The two-particle walk operator in the full 65,536-dimensional virtual space produces non-zero $V_{cb}$ and $V_{ub}$ through correlated double excursions across the code barrier, activating at the double spectral gap $2\Delta = 4$. The ratio $|V_{ub}|/|V_{cb}| \approx 0.1$ matches the experimental value 0.093 with zero fitted parameters.

**3. CKM non-universality prediction** (conjecture): $V_{us}$ is environment-independent (single-particle origin), while $V_{cb}$ is environment-dependent (multi-particle origin), offering a structural explanation for the $V_{cb}$ puzzle.

The immediate computational priorities are: (a) derivation of the generation mass hierarchy from the walk operator dynamics, which would fix the absolute magnitudes of $V_{cb}$ and $V_{ub}$; (b) computation of the Jarlskog invariant $J$ in the two-particle space, which would determine whether the correlated tunnelling produces a CP-violating phase; and (c) explicit evaluation of $|V_{cb}|$ in different hadronic topologies (B-meson vs. $\Lambda_b$ baryon), testing the non-universality prediction against existing data.

## Code and Data Availability

The complete Python implementation of all calculations reported in this paper — the single-particle walk operator (256D), Feshbach projection, supercell zone folding, and two-particle meson walk operator (65,536D sparse) — is publicly available at:

https://github.com/neusym/ckm-lattice

All raw data, eigenvalue spectra, and analysis scripts are archived on Zenodo at:

https://doi.org/10.5281/zenodo.XXXXX

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