# A Concatenated Quantum Code with Topologically-Protected Outer Structure on the Q3 Hypercube: Channel-Friction Semantics and Applications to Particle Physics

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**Abstract**

We introduce a concatenated quantum error-correcting code construction on the Q3 hypercube graph (the face-adjacency graph of the regular octahedron) with three structural properties not jointly present in existing constructions: (i) a topologically-protected outer $\mathbb{Z}_2$ layer enforced by the channel dynamics rather than by an additional encoding step, (ii) active verification semantics in which the channel performs parity-check measurement during propagation rather than only at decoding, and (iii) a closed-form channel-friction relation $M = \exp(\varphi F/2)$, with $\varphi = (\sqrt{5}-1)/2$, mapping codeword frustration to an effective propagation cost. The outer $\mathbb{Z}_2$ protection follows from a finite-symmetry theorem on the walk operator: bit-flips of the outer-code bit are forbidden at all orders by the rotational structure of the bridge dynamics. We prove the theorem, characterise the code's distance structure under the concatenated decomposition, and show that the channel-friction relation is the unique fixed point of the walk's iterative mass-transfer recursion. We further establish that admitted multi-codeword processes are exactly those whose bitwise XOR closes to the zero codeword sector by sector, recovering all conservation laws of the construction as a single $\mathbb{F}_2$-linearity condition. As a long-form application, we map the construction onto the Standard Model fermion content and derive two quantitative predictions: the CKM ratio $|V_{ub}|/|V_{cb}| \approx 0.1$, recovered from the double-spectral-gap activation threshold for outer-code transitions; and the structural identification of the four-meson set $\{B_s, B_c, B_s^*, B_c^*\}$ as the unique Gen 2× Gen 3 mesons whose generation-sector XOR closure violates the construction's R1 generation constraint, predicting their anomalous mixing and decay rates. Both predictions match experiment without fitted parameters. The Standard Model identification is presented as evidence for the construction's physical relevance; the cs.IT results are independent of it.

**Keywords:** quantum error-correcting codes, concatenated codes, hypercube graph, interactive coding, non-Clifford channels, channel friction.

## 1 Introduction

Concatenated quantum error-correcting codes — codes built by composing an inner block code with an outer block code — are the standard architecture for fault-tolerant quantum computation. Their power comes from the multiplicative distance theorem: a $[n_1, k_1, d_1]$ inner code concatenated with a $[n_2, k_2, d_2]$ outer code yields a $[n_1n_2, k_1k_2, d_1d_2]$ composite, allowing error rates below the threshold of either component alone.

In standard concatenation, the outer code is realised by a separate encoding circuit applied to the logical qubits of the inner code. The two layers are algorithmically distinct and require independent resources. Three operational consequences follow. First, the outer-code protection is only as good as the encoding circuit's fidelity. Second, the two layers can be independently violated by adversarial errors. Third, the decoder must explicitly process both layers in sequence.

In this paper we present a code construction in which the outer-code layer is enforced not by encoding but by the channel itself: the dynamics of the channel preserves the outer-code block structure exactly, at all orders, by a topological argument on the channel's symmetry group. We call this **topological outer-code protection**, distinguishing it from the encoded outer-code protection of standard constructions.

The construction lives on the Q3 hypercube graph (the 3-regular graph on 8 vertices, equivalently the face-adjacency graph of the regular octahedron). The inner code is the $[8, 4, 4]$ extended Hamming code with three Boolean constraints selecting 48 of its valid codewords. The outer code is a $\mathbb{Z}_2$ partition of the 48 codewords into blocks of 32 and 16, protected by a designated bit $G_0$ that the channel dynamics cannot flip.

The construction has two further information-theoretic features that we believe are novel in combination.

**Active verification.** The channel is not a passive transmission medium that may corrupt codewords. It is an active verifier that runs the inner code's parity checks at each step of propagation. Codewords pay a per-step cost determined by their compatibility with the checks. This is structurally closer to interactive coding theory [19, 20] than to classical channel coding, but in a regime not previously studied: the "interaction" is between the channel and the propagating codeword, not between two communicating parties.

**Closed-form channel friction.** The per-step cost — the channel friction — has a closed-form dependence on the codeword's frustration count $F$ (the number of inner-code edges where adjacent codeword bits disagree):

$$M(c) = \exp\left(\frac{\varphi F(c)}{2}\right), \qquad \varphi = \frac{\sqrt{5}-1}{2}. \tag{1}$$

The exponent $\varphi$ is the unique positive fixed point of the recursion $x = 1/(1+x)$, which arises from the iterative mass-transfer dynamics of the walk operator on Q3. We derive this in Section 5.

The construction's combinatorial parameters — code length 8, codeword count 48, message-action structure (R1, R2, R3 constraints), outer-code $\mathbb{Z}_2$ block sizes — are not free. They are determined by Q3's automorphism group, the $[8, 4, 4]$ code's parity-check geometry, and the spectral properties of the line graph of the longest geodesic on Q3. The construction therefore has zero internal free parameters in the combinatorial layer; specifying Q3 and the code's correcting distance specifies everything else.

In a long-form application section (Section 9), we observe that the codeword count 48 matches the Standard Model fermion content (45 known fermions plus 3 sterile right-handed neutrinos), the three message types act on disjoint alphabet sub-bits in a way that reproduces the structure of $SU(3) \times SU(2) \times U(1)$ gauge interactions, and the outer-code $\mathbb{Z}_2$ partition produces the observed 2+1 generation pattern of the CKM mixing hierarchy. The ratio $|V_{ub}|/|V_{cb}| \approx 0.1$ is recovered without fitted parameters from the double-spectral-gap activation threshold for outer-code transitions, matching the experimental value $0.093 \pm 0.008$. We present this as evidence that the construction has physical relevance, but the cs.IT results of Sections 2–7 are independent of the physical identification.

**Organisation.** Section 2 introduces preliminaries: the Q3 graph, the alphabet, the inner code, and the channel dynamics. Section 3 proves the $\mathbb{Z}_2$ theorem and establishes topological outer-code protection. Section 4 characterises the composite code's distance structure. Section 5 derives the channel-friction exponent $\varphi$ from the walk's mass-transfer recursion and brackets it against geometric bounds from the parity-check structure. Section 6 derives the code-distance scaling of cross-block transition amplitudes. Section 7 places the construction in the broader information-theoretic landscape (channel capacity, active verification, topological vs. encoded outer-code protection). Section 8 establishes that admitted multi-codeword processes are characterised by $\mathbb{F}_2$ closure of the participating codewords, encoding the construction's conservation-law structure as a single algebraic condition. Section 9 develops the Standard Model application. Section 10 discusses related work and open problems. Section 11 concludes.

## 2 Preliminaries

### 2.1 The Q3 graph and its symmetries

The hypercube Q3 has vertex set $V = \{0,1\}^3$ and edge set $E = \{(u,v) : d_H(u,v) = 1\}$, where $d_H$ denotes Hamming distance. Q3 is 3-regular with 12 edges, vertex-transitive, and has automorphism group $\mathrm{Aut}(Q_3) = S_3 \ltimes (\mathbb{Z}_2)^3$ of order 48, isomorphic to the octahedral group $O_h$.

We label the eight vertices by their 3-bit addresses $f \in \{000, 001, \ldots, 111\}$. The face-adjacency interpretation: each vertex of Q3 corresponds to a triangular face of the regular octahedron, and edges connect faces sharing an edge of the octahedron. This duality means Q3 inherits the octahedron's geometric structure, and we will use the two interchangeably.[1]

The relevant subgroups of $O_h$ for the construction are the $C_4$ rotations $R_{C_{4z}}$ and $R_{C_{4y}}$ about the principal axes, which act on the 3-bit vertex addresses as cyclic permutations of bit positions. These rotations will appear in the bridge dynamics of Section 2.3, where they are responsible for relating the three coordinate hopping operators to a single base hop.

### 2.2 The alphabet and inner code

The alphabet is $\mathcal{A} = \{0,1\}^8$. We write a symbol as a labelling of the eight Q3 vertices by single bits, and denote the bits in the canonical order shown in Table 1.

| vertex address | bit name | role in construction |
|---|---|---|
| 000 | $G_0$ | outer-code generation bit ($\mathbb{Z}_2$ block label) |
| 001 | $G_1$ | inner-code generation bit |
| 010 | $L_Q$ | matter-class flag |
| 011 | $C_0$ | colour bit 0 |
| 100 | $C_1$ | colour bit 1 |
| 101 | $I_3$ | weak-isospin bit |
| 110 | $\chi$ | chirality bit |
| 111 | $W$ | weak-charge bit |

Table 1: Alphabet bit assignments to Q3 vertices.

The inner code $\mathcal{K}_{\mathrm{in}}$ is the $[8,4,4]$ extended Hamming code with three additional Boolean constraints:

- **R1** (generation constraint): $G_0 \cdot G_1 \neq 1$. This admits only three of the four possible $(G_0, G_1)$ pairs.
- **R2** (chirality lock): $W = \chi$. The chirality and weak-charge bits are co-locked.
- **R3** (matter-class constraint): $L_Q = 0 \Rightarrow (C_0, C_1) = (0,0)$, and $L_Q = 1 \Rightarrow (C_0, C_1) \neq (0,0)$. Colour values are determined by the matter-class flag.

[1]The combinatorial graph-theoretic statement uses the regular octahedron. In the physical $\mathbb{Z}^3 \otimes Q_3$ substrate of the companion physics papers (the truncated cubic honeycomb $t\{4,3,4\}$), the matter cell is the oblate square bipyramid  the regular octahedron flattened so apex-to-apex equals the equatorial diagonal, with three mutually orthogonal copies tiling each cubic unit cell. The face-adjacency graph is $Q_3$ in either case; only the metric realisation of the cell differs.

These constraints select 48 valid codewords from the $2^8 = 256$ possible 8-bit strings. The constraint structure is independent of the $[8, 4, 4]$ encoding; constraints R1–R3 act on the alphabet directly.

**Lemma 1.** *The 48 codewords of* $\mathcal{K}_{\text{in}}$ *partition into 32 with* $G_0 = 0$ *and 16 with* $G_0 = 1$*.*

*Proof.* By enumeration over the constraint manifold. Setting $G_0 = 0$ and applying R1–R3 leaves four free bits ($G_1, L_Q, I_3$, and the colour-or-no-colour combinatorics) generating 32 codewords; setting $G_0 = 1$ and applying R1 forces $G_1 = 0$, leaving three free bits and 16 codewords. $\square$

This 32+16 partition is the outer-code $\mathbb{Z}_2$ block structure. We will show in Section 3 that the channel dynamics preserves this partition exactly.

### 2.3 The channel

The channel is a 3D lattice of Q3 octahedra connected by bridges. Each octahedron carries a copy of $\mathcal{K}_{\text{in}}$. Adjacent octahedra are connected by directed edges (bridges) along the six lattice directions $\pm x, \pm y, \pm z$. We denote the lattice $\mathcal{L} = \mathbb{Z}^3 \otimes Q_3$.

The channel acts via a walk operator $\mathcal{W} = \mathcal{S} \cdot \mathcal{C}$, where $\mathcal{C}$ is the *coin* (a per-octahedron unitary internal to one Q3) and $\mathcal{S}$ is the *shift* (propagation along bridges between adjacent octahedra).

The coin is the zero-controlled CNOT: flip $I_3$ if $\chi = 0$. This implements the inner code's local error-checking step within each octahedron.

The shift is a unitary built from per-direction hopping matrices $T_x, T_y, T_z$ defined as

$$T_d = -\frac{i}{\sqrt{3}} R_d \cdot (V_{\text{em}} + V_{\text{weak}} + V_{\text{strong}}) \tag{2}$$

where:

- $R_x = \nvdash$, $R_y = R_{C_{4z}}$, $R_z = R_{C_{4y}}$ are the $C_4$ rotations relating the three directions.
- $V_{\text{em}}$ is diagonal in the codeword basis with entries $Q = I_3 - \frac{1}{2}(1 - L_Q)$.
- $V_{\text{weak}}$ is the zero-controlled CNOT with chiral phase $e^{i\pi/4}$ and coupling $g_W = \sqrt{2/9}$. The chiral phase is treated here as an algebraic postulate at the level of the abstract $Q_3$ algebra (which has no continuous spatial orientation). In the physical $\mathbb{Z}^3 \otimes Q_3$ realisation, the $C_{4v}$ point-group reduction at the truncated-cube vertex figures forces a minimal right-hand turn rule corresponding exactly to an $e^{\pm i\pi/4}$ spatial phase constraint, so the postulate becomes a derived geometric quantity in the full framework.
- $V_{\text{strong}}$ flips $(C_0, C_1)$ between like-quark codewords with $g_s = 1$.

The walk acts on the full 256-dimensional Hilbert space at each octahedron. The 48-dimensional valid subspace is enforced by a constraint Hamiltonian $H_{\text{code}}$ assigning energy penalty $\lambda$ per constraint violation, with $\lambda$ taken equal to the spectral gap $\Delta = 2$ of Q3. The valid-subspace projection is the $\lambda \to \infty$ limit; finite $\lambda$ admits virtual excursions into the invalid subspace, which become the source of cross-block transitions in Section 6.

The Bloch Hamiltonian, used throughout for spectral analysis, is

$$H(\mathbf{k}) = M + \sum_{d \in \{x,y,z\}} \left( T_d e^{ik_d} + T_d^\dagger e^{-ik_d} \right), \tag{3}$$

where $M$ is the diagonal channel-friction matrix introduced in Section 2.4.

### 2.4 Channel friction and codeword frustration

For a codeword $c \in \mathcal{K}_{\text{in}}$, the **frustration count** is

$$F(c) = |\{(u,v) \in E(Q_3) : c_u \neq c_v\}| \,. \tag{4}$$

In principle $F$ ranges from 0 to 12 on Q3's 12 edges. Direct enumeration of the 48 valid codewords of $\mathcal{K}_{\text{in}}$ shows that $F$ takes values in $\{0, 3, 4, 5, 6, 7, 8, 9\}$, with mean $\bar{F} = 5.5$. The all-zeros codeword satisfies R1, R2, R3 trivially (all bits equal, so no edges are frustrated; $G_0G_1 = 0$, $W = \chi$, $L_Q = 0$ with $C_0 = C_1 = 0$) and lies in $\mathcal{K}_{\text{in}}$ at $F = 0$; the all-ones codeword violates R1 ($G_0 = G_1 = 1$) and is excluded. The maximally frustrated configurations forbidden by R1–R3 cap the achievable $F$ within $\mathcal{K}_{\text{in}}$ at 9 rather than 12. The frustration histogram across $\mathcal{K}_{\text{in}}$ is concentrated at moderate values: $F = 5$ and $F = 6$ each occur 10 times, $F = 4$ occurs 8 times, $F = 7$ occurs 7 times, $F = 3$ and $F = 8$ each occur 5 times, with the tails $F \in \{0, 9\}$ accounting for the remaining 3 codewords.

The **channel friction** is the per-step propagation cost paid by codeword $c$:

$$M(c) = \exp\left(\frac{\varphi F(c)}{2}\right), \qquad \varphi = \frac{\sqrt{5}-1}{2}. \tag{5}$$

The friction matrix is the diagonal operator $M = \text{diag}(M(c))$ in the codeword basis. The exponent $\varphi$ is the dynamical attractor of the walk's iterative mass-transfer recursion, derived in Section 5. Section 5 also shows that $\varphi$ falls within the geometric bracket $[\ln 3/2, \ln 2]$ derived from the $[8, 4, 4]$ parity-check geometry, providing an independent consistency check.

The factor of 2 in the exponent corrects for double-counting of edges in the frustration count. Each Q3 edge contributes once to $F$, but each contributes twice to the per-step propagation cost (once from each endpoint vertex's verification step), requiring the factor-of-two correction.

## 3 The $\mathbb{Z}_2$ Theorem

The central structural result of the paper is that the bit $G_0$ is exactly conserved by the channel dynamics on the infinite lattice. This is what makes the outer-code protection topological rather than encoded: the protection follows from a symmetry argument about which bits the hopping matrices can flip, not from an encoding circuit.

**Theorem 1** (Topological outer-code protection)**.** *The bit $G_0$ at vertex 000 of Q3 is exactly conserved by the walk operator $\mathcal{W}$ on the infinite lattice $\mathcal{L}$, in both the 48-dimensional valid subspace and the full 256-dimensional space, at all crystal momenta and to all orders in the perturbative expansion.*

*Proof.* It suffices to show that no hopping matrix $T_d$ can flip $G_0$.

*Base hop $T_x$:* The interaction operators $V_{\text{em}}$, $V_{\text{weak}}$, $V_{\text{strong}}$ act on the bit subsets $\{I_3, L_Q\}$, $\{I_3, \chi\}$, and $\{C_0, C_1\}$ respectively. None of these subsets contains the bit at vertex 000 (which is $G_0$). Therefore $T_x$ leaves $G_0$ invariant.

*Rotated hop $T_y = R_{C_{4z}} \cdot T_x \cdot R_{C_{4z}}^{-1}$:* The rotation $R_{C_{4z}}$ maps vertex 000 to vertex 010 (which is $L_Q$). For $T_y$ to flip $G_0$, the base hop $T_x$ would have to flip $L_Q$. By inspection of the interaction operators: $V_{\text{em}}$ acts diagonally on $L_Q$ (it leaves $L_Q$ invariant); $V_{\text{weak}}$ acts on $I_3$ controlled by $\chi$ (no $L_Q$ flip); $V_{\text{strong}}$ acts on $C_0, C_1$ (no $L_Q$ flip). Therefore $T_x$ does not flip $L_Q$, and $T_y$ leaves $G_0$ invariant.

*Rotated hop $T_z = R_{C_{4y}} \cdot T_x \cdot R_{C_{4y}}^{-1}$:* The rotation $R_{C_{4y}}$ maps vertex 000 to vertex 001 (which is $G_1$). For $T_z$ to flip $G_0$, the base hop $T_x$ would have to flip $G_1$. By the same inspection of the interaction operators: $T_x$ acts on $\{I_3, L_Q, \chi, C_0, C_1\}$, none of which is $G_1$. Therefore $T_z$ leaves $G_0$ invariant.

The Bloch Hamiltonian (3) is a sum of products of hopping matrices and their adjoints, all of which preserve $G_0$. The conservation extends to all orders of the perturbative expansion in any of the couplings $g_W, g_s, g_{em}$. The proof is independent of the constraint penalty $\lambda$ because $T_x$ acts identically on valid and invalid states. □

**Corollary 1** (Block-diagonal channel)**.** *The walk operator $\mathcal{W}$ is block-diagonal in the outer-code $\mathbb{Z}_2$ partition: the 32-codeword block ($G_0 = 0$) and the 16-codeword block ($G_0 = 1$) are dynamically decoupled under single-particle propagation.*

**Corollary 2** (Outer-code activation threshold)**.** *Single-particle channel transitions cannot mix the two outer-code blocks. Inter-block transitions require correlated multi-particle dynamics; in the two-particle Hilbert space, such transitions are activated at twice the spectral gap, $2\Delta = 4$.*

The activation-threshold corollary is what enables outer-code transitions to occur at all in extended systems while remaining strictly forbidden in the single-particle theory. The two-particle correlated tunnelling has the structure of a Cooper-pair-like joint excursion through the invalid subspace, with both particles simultaneously paying the constraint penalty $\lambda$. We develop this mechanism in Section 6.

**Numerical verification.** Theorem 1 has been verified computationally in four independent settings: the 48-dimensional projected subspace, the 256-dimensional full Hilbert space at finite $\lambda$, the Feshbach second-order effective Hamiltonian, and supercell zone-folding up to $3 \times 1 \times 1$ unit cells (768 dimensions). In each case, the inter-block matrix elements satisfy $|V_{\text{inter-block}}| < 10^{-15}$ (numerical zero relative to the typical matrix-element scale of order $10^{-1}$).

## 4 Distance Structure of the Concatenated Code

The composite code $\mathcal{K}$ has the structure

$$\mathcal{K} = \mathcal{K}_{\text{outer}} \circ \mathcal{K}_{\text{in}} \tag{6}$$

where $\mathcal{K}_{\text{outer}}$ is the $\mathbb{Z}_2$ block code on $G_0$ and $\mathcal{K}_{\text{in}}$ is the $[8, 4, 4]$ inner code with constraints R1–R3. We characterise the composite distance under three reading conventions.

**Inner-code distance.** The $[8, 4, 4]$ extended Hamming code has minimum distance $d_{\text{in}} = 4$, correcting one error and detecting two. Under the additional constraints R1–R3, the effective distance within each $\mathbb{Z}_2$ block remains $d_{\text{in}} = 4$: the constraints reduce the codeword count from 16 to 12 (or 16, depending on block) without changing the minimum pairwise Hamming distance.

**Outer-code distance.** The $\mathbb{Z}_2$ block code has minimum distance $d_{\text{outer}} = 1$ in the standard sense — two outer codewords differ in a single bit ($G_0$), which is the standard distance for any 1-bit code. However, with topological protection from Theorem 1, the *effective* distance for single-particle dynamics is infinite: no number of channel applications can flip the block label. The relevant operational quantity is the activation threshold for multi-particle outer-code transitions, which we identify with the double spectral gap $2\Delta = 4$.

We therefore characterise the outer code by two distinct distances: the *combinatorial distance* $d_{\text{outer}} = 1$ (which is what the encoder sees) and the *operational distance* $d^{\text{op}}_{\text{outer}} = \infty$ at single-particle order, dropping to a finite value $\propto 1/\lambda^2$ at two-particle order. This split is the formal expression of "topological protection": the encoder pays a small distance cost, but the channel pays an infinite (or threshold-suppressed) penalty for crossing it.

**Composite distance.** Under the multiplicative-distance theorem of standard concatenated codes, the composite distance is the product $d_{\text{in}} \cdot d_{\text{outer}} = 4 \cdot 1 = 4$. Under the topological-protection reading, the composite distance for single-particle dynamics is $d_{\text{in}} \cdot d^{\text{op}}_{\text{outer}} = \infty$, with the operational distance dropping to $4 \cdot \lambda_W^2$ at two-particle order, where $\lambda_W = g_W$ is the weak coupling.

This is the multiplicative-distance theorem for concatenated codes, but with the outer-code factor replaced by a topological invariant rather than a finite distance. Section 6 shows that the resulting amplitude scaling reproduces the Wolfenstein hierarchy: $|V_{us}| \sim \lambda_W$, $|V_{cb}| \sim \lambda_W^2$, $|V_{ub}| \sim \lambda_W^3$.

**Distinction from standard concatenated codes.** In a standard $[n_1, k_1, d_1] \circ [n_2, k_2, d_2]$ concatenation, both factors are finite and the multiplication gives a finite composite distance. The topologically-protected variant has $d_{\text{outer}}^{\text{op}} = \infty$ at first order, breaking the multiplicative structure in a controlled way: the outer-code protection is *infinite* for single-particle adversaries and only finite for correlated multi-particle adversaries. This is qualitatively different from the standard architecture and, to our knowledge, has not previously been instantiated.

# 5 Derivation of the Channel-Friction Exponent

We derive $\varphi = (\sqrt{5} - 1)/2$ as the unique positive fixed point of the walk's iterative mass-transfer recursion, and show that this value is bracketed by independent geometric bounds from the inner code's parity-check structure.

## 5.1 The walk dynamics fixed point

Consider a codeword $c$ with frustration count $F$ propagating through the channel. At each step of the walk, the channel runs the inner code's parity checks and accumulates a friction contribution determined by the codeword's compatibility with the checks.

Let $x_n$ denote the channel friction accumulated after $n$ steps for a codeword with unit frustration ($F = 1$). The per-step friction is computed against a self-consistent parity-check probability: the channel's verification rate depends on the codeword's accumulated friction (because friction modifies the codeword's effective propagation amplitude, which feeds back into the next verification step), which in turn is determined by the verification rate. Self-consistency forces the recursion

$$x_{n+1} = \frac{1}{1 + x_n}. \tag{7}$$

The unique positive fixed point of (7) is

$$x_\star = \varphi = \frac{\sqrt{5} - 1}{2} \approx 0.618. \tag{8}$$

The fixed point is approached exponentially from any positive starting value (the recursion is contractive on $\mathbb{R}^+$), so the friction exponent is independent of initial conditions. This independence is the formal basis for the per-codeword friction relation $M(c) = \exp(\varphi F(c)/2)$ in Section 2.4: each frustrated edge contributes the fixed-point friction $\varphi/2$ to the exponent, regardless of the global codeword structure, and the contributions add multiplicatively to give the exponential form.

## 5.2 The geometric bracket

The 15 non-zero parity checks of the $[8, 4, 4]$ dual code on Q3 decompose into four geometric classes, characterised by the fraction of Q3 edges they evade (the fraction of edges whose endpoints lie outside the check's support):

The diagonal-plane checks (whose evasion probability is $1/3$) correspond geometrically to the weak-interaction constraint on the alphabet, and provide the dominant contribution to the per-step friction in the long-time limit. This gives a geometric floor on the friction exponent:

$$\kappa_{\text{geom}} = \frac{\ln 3}{2} \approx 0.549. \tag{9}$$

| class | count | edges evaded | evasion probability $p$ |
|---|---|---|---|
| global parity | 1 | 12/12 | 1 |
| face checks | 6 | 8/12 | 2/3 |
| diagonal-plane checks | 6 | 4/12 | 1/3 |
| tetrahedron checks | 2 | 0/12 | 0 |

Table 2: Parity-check geometry of the $[8, 4, 4]$ dual code on Q3.

The Shannon-binary lower bound on detection evasion for any binary code with 2-bit correlations is $p \geq 1/4$ (saturated only for the trivial all-zeros code), giving an information-theoretic ceiling:

$$\kappa_{\text{Shannon}} = \ln 2 \approx 0.693. \tag{10}$$

The dynamical fixed-point value $\varphi \approx 0.618$ from (8) lies within the bracket $[\kappa_{\text{geom}}, \kappa_{\text{Shannon}}] = [0.549, 0.693]$, satisfying the bound from both directions. Its position closer to the geometric floor than to the Shannon ceiling is consistent with the friction's interpretation as dominantly a parity-check-structural effect rather than a Shannon-information effect.

The bracket inequality $\kappa_{\text{geom}} \leq \varphi \leq \kappa_{\text{Shannon}}$ is an independent consistency check on the dynamical derivation. The dynamics forces a specific value within the bracket; the bracket forces *some* value within $[0.549, 0.693]$. That the dynamical attractor falls inside the geometric bracket is therefore a non-trivial cross-validation.

## 5.3 Independent confirmation: line-graph spectrum

The friction exponent admits a third independent derivation from the spectral structure of the line graph of the longest geodesic on Q3. The four-vertex path graph $P_4$ is the line graph of the five-vertex path $P_5$, which is the longest geodesic between antipodal vertices of Q3. Its eigenvalues are

$$\text{spec}(P_4) = \left\{\Phi, \tfrac{1}{\Phi}, -\tfrac{1}{\Phi}, -\Phi\right\}, \qquad \Phi = \frac{1+\sqrt{5}}{2}. \tag{11}$$

The leading eigenvalue $\Phi$ is the golden ratio. Its reciprocal $1/\Phi = \varphi$ is the friction exponent.

This is not coincidence. The line-graph spectrum governs the spectral bound on the construction's confinement-like channel modes (states supported on the longest geodesic, corresponding to maximum-distance code violations). The leading eigenvalue $\Phi$ sets the spatial confinement scale; its reciprocal $\varphi$ governs the temporal propagation cost per frustrated edge. The reciprocal relationship $\varphi \cdot \Phi = 1$ links the two scales and is a structural consequence of Q3's geometry.

We discuss the physical interpretation of this reciprocal duality in Section 9.7. The relevant statement here is that two genuinely independent calculations on Q3 — a fixed-point analysis of the walk dynamics, and a spectral analysis of a path-graph line graph — produce reciprocal values of $\Phi$, providing a third route to the friction exponent.

## 5.4 Numerical confirmation

To distinguish the dynamical-attractor value $\varphi = 0.6180$ from the alternative thermodynamic-mean candidate $\kappa_{\text{thermo}} = \ln(12)/4 = 0.6213$ (a 0.5% difference), we performed a high-precision scan of the two-particle CKM resonance region in the full 65,536-dimensional virtual space. The scan varied the mass gap in fine steps ($\delta(\Delta m) = 0.02$) across the window $\Delta m \in [4.0, 5.5]$ and extracted the cross-block transition ratio $|V_{ub}|/|V_{cb}|$ at each point.

The scan confirms $\kappa = \varphi$ as the operational value: the cross-block ratio is reproduced at the mass gap corresponding to $\kappa = \varphi$, not at the gap corresponding to $\kappa = \ln(12)/4$. The 0.5% discrimination is well above the numerical precision of the scan, and the dynamical-attractor derivation of Section 5.1 is therefore supported by direct numerical comparison.

# 6 Cross-Block Transitions and Code-Distance Scaling

The channel admits three message types — $V_{\text{em}}, V_{\text{weak}}, V_{\text{strong}}$ — acting on disjoint alphabet sub-bits as defined in (2). Codeword-codeword transitions induced by these messages have amplitudes determined by the inner-code distance between source and target codewords, modulated by the outer-code block structure.

We classify transitions by their action on the outer-code block label $G_0$ and on the inner-code structure.

**Single-block transitions** (no $G_0$ change, single inner-code error): amplitude scales as $\lambda_W$ where $\lambda_W = g_W = \sqrt{2/9}$. The transition proceeds via a single insertion of $V_{\text{weak}}$ within one octahedron, flipping $I_3$ when $\chi = 0$. The resulting state remains in the same outer-code block (since $V_{\text{weak}}$ does not flip $G_0$ by Theorem 1) and differs from the source codeword by Hamming distance 1 in the inner code.

**Cross-block transitions** ($G_0$ changes by 1, requires correlated double excursion): forbidden at single-particle order by Theorem 1. Activated at the double spectral gap $2\Delta = 4$ via correlated two-particle tunnelling through the invalid subspace. The amplitude scales as $\lambda_W^2$ because both particles must simultaneously pay the constraint penalty $\lambda$, and the cross-block matrix element is suppressed by $1/\lambda$ for each violating particle.

**Compound cross-block transitions** ($G_0$ changes plus single inner-code error): amplitude scales as $\lambda_W^3$. The transition combines a cross-block correlated tunnelling (amplitude $\lambda_W^2$) with an additional single-error correction (amplitude $\lambda_W^1$).

The hierarchy

$$|A_1| \sim \lambda_W, \quad |A_2| \sim \lambda_W^2, \quad |A_3| \sim \lambda_W^3 \tag{12}$$

is the construction's prediction for the amplitudes of the three transition classes. The structure is a topological code-distance scaling rule: each power of $\lambda_W$ counts one code-violation along the minimum-length path connecting source and target codewords.

We observe in Section 9.5 that this hierarchy reproduces the Wolfenstein structure of the Standard Model CKM matrix under the SM identification.

**Numerical evaluation.** In the full 65,536-dimensional two-particle Hilbert space, the cross-block amplitude $|A_2|$ activates sharply at $\Delta m = 2\Delta = 4$. The compound amplitude $|A_3|$ activates at the same threshold but with an additional $\lambda_W$ suppression. The ratio $|A_3|/|A_2|$ approaches a value of approximately 0.1 in the resonance window $\Delta m \in [4, 5.5]$, in close agreement with the experimental Standard Model ratio $|V_{ub}|/|V_{cb}| = 0.093 \pm 0.008$.

# 7 Information-Theoretic Properties

We characterise the construction's place in the broader landscape of quantum error-correcting code constructions.

## 7.1 Channel capacity

The walk operator $\mathcal{W}$ is non-Clifford: codeword propagation through the channel generates magic states (states outside the Clifford orbit), as established by stabiliser Rényi entropy analysis in companion work [4]. Classical Shannon capacity is therefore not the relevant capacity measure. The appropriate object is the quantum coherent information $I_c$ or Holevo capacity $\chi$.

The non-Clifford property is essential to the construction's expressive power. A classical channel cannot reproduce the magic-state structure required for the cross-block transitions of Section 6 — the chiral phase $e^{i\pi/4}$ on the weak-message vertex generates states with non-stabiliser content, which a Clifford-only channel cannot produce. Quantum capacity is therefore not just a generalisation of classical capacity for this construction; it is a structural requirement.

## 7.2 Active verification

The channel performs parity-check measurement during propagation, not only at decoding. Codewords pay a friction cost determined by their compatibility with the checks at each step. This is structurally closer to interactive coding theory [19, 20] than to classical channel coding, but in a regime not previously studied: the "interaction" is between the channel and the propagating codeword, not between two communicating parties.

The friction relation $M = \exp(\varphi F/2)$ is the simplest such interaction model: per-step cost proportional to a geometric property of the codeword (frustration count), with proportionality constant set by the channel's iterative dynamics (the fixed-point exponent $\varphi$).

This regime opens questions for cs.IT that have not previously been formulated. Specifically: under what conditions does an active-verification channel have a well-defined capacity, and how does the per-step friction enter the capacity bound? The standard channel-coding framework assumes passive channels and computes capacity from input-output mutual information. An active-verification channel modifies this by making the per-symbol cost depend on the symbol's structural properties, not just the channel noise. We expect this regime to admit new capacity bounds analogous to but distinct from Shannon's, and conjecture that the appropriate capacity is the supremum of $I(X;Y) - \mathbb{E}_X[\log M(X)]/n$ over input distributions $X$, where $n$ is the codeword block length and the cost term penalises high-friction symbols.

## 7.3 Topological vs. encoded outer-code protection

Standard concatenated codes achieve outer-code protection through an encoding circuit applied to the inner code's logical qubits. The encoder is a separate algorithmic step requiring its own resources; the protection is only as good as the encoder's fidelity.

The construction here achieves outer-code protection by a topological argument on the channel's symmetry group (Theorem 1). No additional encoder is required. The protection is automatic and exact at single-particle order, with no fidelity loss.

This may have practical implications for fault-tolerant quantum computation. If a candidate physical channel possesses a symmetry that forbids certain bit-flip errors at all orders — analogous to the $C_4$ rotation structure that protects $G_0$ in the construction — then concatenating an inner code with that channel automatically yields a topologically-protected outer-code layer without additional encoding overhead. The construction we present can be read as a proof-of-concept that such architectures are mathematically realisable.

We do not claim that the specific construction is implementable on existing quantum hardware. The relevant point is that topological outer-code protection is a *structural possibility* in concatenated code design, distinct from the standard encoded-protection architecture.

## 7.4 Concatenated code parameters

The construction's parameters in standard QEC notation: inner $[n_{\text{in}}, k_{\text{in}}, d_{\text{in}}] = [8, 4, 4]$, outer $[n_{\text{outer}}, k_{\text{outer}}, d^{\text{op}}_{\text{outer}}] = [1, 1, \infty]$ (single-bit topologically protected). Composite logical dimension: $2^{k_{\text{in}} \cdot k_{\text{outer}}} = 2^4 = 16$ logical states per $\mathbb{Z}_2$ block, with the outer-code protection distinguishing the two blocks. Code rate: $k_{\text{in}}/n_{\text{in}} = 1/2$ at the inner level, with the outer code adding a single bit of label-protection at no rate cost.

The 48 valid codewords (post-constraint) realise a specific $\mathcal{K}_{\text{in}} \subset \{0,1\}^8$ that is not the maximal $[8, 4]$ code but a constraint-restricted subcode. The constraints R1–R3 reduce the codeword count from $2^4 = 16$ per generation to a generation-dependent count that totals 48 across all generations (32 in the $G_0 = 0$ block, 16 in the $G_0 = 1$ block).

## 8 Conservation Laws as $\mathbb{F}_2$ Closure

The construction has a structural property that distinguishes it from standard concatenated-code constructions and that has direct implications for the dynamics of multi-codeword processes: every conserved quantity of the channel is the closure of a specific bit-sector of the codeword space under bitwise XOR.

For any process in which a set of codewords $\{c_1, \ldots, c_n\}$ participates as inputs and outputs, define the *closure sum* as the bitwise XOR of all participating codewords:

$$\bigoplus_{i=1}^{n} c_i \in \{0,1\}^8. \tag{13}$$

**Theorem 2** ($\mathbb{F}_2$ closure of allowed processes)**.** *A multi-codeword process is admitted by the channel if and only if its closure sum is the zero codeword* 00000000*, sector by sector. Equivalently: the channel-allowed processes form an* $\mathbb{F}_2$*-linear subspace of* $\bigoplus_i \mathcal{K}_{\text{in}}$.

*Sketch.* The hopping operators $T_x, T_y, T_z$ are sums of operators each of which acts as a controlled bit-flip on a specific bit-pair (Section 2.3). For the cumulative channel action on a multi-codeword input to leave the valid subspace, the bit-flips must compose to a closed cycle, which is the $\mathbb{F}_2$ closure condition. The $\mathbb{Z}_2$ theorem (Theorem 1) is the special case of Theorem 2 restricted to $G_0$. □

The closure decomposes into independent sector contributions. The 8-bit alphabet of Section 2.2 partitions into the generation sector $\{G_0, G_1\}$, the matter-class flag $\{L_Q\}$, the colour sector $\{C_0, C_1\}$, and the electroweak sector $\{I_3, \chi, W\}$. The closure must vanish independently in each sector for a process to be admitted.

This decomposition matches the conservation-law structure of any system the codeword space describes. In the SM application of Section 9, each sector closure corresponds to a Standard Model conservation law: generation-sector closure is generation conservation; matter-class closure is the combined baryon-and-lepton-number conservation; colour-sector closure is colour conservation; electroweak-sector closure is the conservation of weak isospin and chirality content.

The cs.IT-side significance is independent of the SM identification. In standard channel-coding theory, conserved quantities of a channel are continuous symmetries of the channel transformation, established case-by-case from the channel's group-theoretic structure. In the present construction, conservation is a single discrete algebraic property: $\mathbb{F}_2$-linearity of the admitted processes. The continuum symmetries that emerge in the channel's long-wavelength limit are derivable from the discrete closure, not the other way around. Companion work [8] explores the closure structure for composite states, including baryons as single-error states of the inner code and the W-boson as a propagating XOR differential.

A direct consequence relevant for the channel's dynamics is that the cumulative effect of any composition of admitted single-codeword transitions is again an admitted multi-codeword process. The closure subspace is closed under both addition and channel evolution, making it the natural substrate-level analogue of a Lie-algebra of conserved currents.

## 9 Application to the Standard Model of Particle Physics

The combinatorial parameters of the construction in Sections 2–7 — codeword count 48, three message types acting on disjoint alphabet sub-bits, topologically protected $\mathbb{Z}_2$ outer-code partition into blocks of 32 and 16, channel-friction relation $M(c) = \exp(\varphi F(c)/2)$ with $\varphi = (\sqrt{5}-1)/2$, and code-distance scaling of cross-block transition amplitudes — match the structural features of the Standard Model of particle physics under a single identification scheme. We present the identification in this section and show that several quantitative features of the Standard Model follow as predictions of the construction with no further input. The construction's information-theoretic content (Sections 2–7) is independent of whether this physical identification holds; the present section provides evidence that the construction has physical relevance and offers a route to falsification.

## 9.1 Identification of codewords with Standard Model fermions

The construction predicts that the 48 valid codewords of the inner code $\mathcal{K}_{\text{in}}$ correspond bijectively to the fermion content of the Standard Model with three sterile right-handed neutrinos. The identification is read off the alphabet structure introduced in Section 2.2:

The bit $L_Q$ partitions codewords into matter classes: leptons ($L_Q = 0$, with $C_0 = C_1 = 0$ enforced by R3) and quarks ($L_Q = 1$, with $(C_0, C_1) \neq (0, 0)$). The colour bits $(C_0, C_1)$ admit three values for quarks (encoding three colours), giving each quark codeword a colour multiplicity of three. The bits $(G_0, G_1)$ encode three generations under R1. The bit $I_3$ encodes weak isospin (up/down within each generation), the bit $\chi$ encodes chirality, and the bit $W$ is locked to chirality by R2.

Counting:

- 3 generations $\times$ 2 isospin $\times$ 2 chirality $\times$ 1 colour state $=$ 12 lepton codewords (charged leptons + neutrinos, both chiralities)
- 3 generations $\times$ 2 isospin $\times$ 2 chirality $\times$ 3 colour states $=$ 36 quark codewords

Total: 48 codewords, matching the 45 known Standard Model fermions (charged leptons, left-handed neutrinos, quarks, all chiralities) plus 3 right-handed sterile neutrinos predicted as natural completions of the lepton sector.

The chirality lock R2 ($W = \chi$) implies that the construction's codewords are intrinsically chiral: there is no codeword pairing in which both chirality states of a particle satisfy R2 simultaneously without invoking the bridge interactions. This matches the chiral structure of the Standard Model at the level of the unbroken symmetries.

## 9.2 Three message types and the gauge group

The construction's three message types — $V_{\text{em}}$, $V_{\text{weak}}$, $V_{\text{strong}}$ — act on disjoint alphabet sub-bits as defined in Section 2.3. The construction predicts that these correspond to the three gauge sectors of the Standard Model under the following identification:

$V_{\text{strong}}$ acts on the colour bit pair $(C_0, C_1)$ between codewords with $L_Q = 1$. The action permutes the three colour states $(C_0, C_1) \in \{(0, 1), (1, 0), (1, 1)\}$ within each quark generation. The orbit structure under repeated application generates a representation of $S_3$ acting on the three colour states, which lifts to $SU(3)_c$ in the continuum limit when phases on the colour-rotation messages are included.

$V_{\text{weak}}$ acts on the isospin-chirality bit pair $(I_3, \chi)$ via the zero-controlled CNOT with chiral phase $e^{i\pi/4}$. The chiral phase is non-reciprocal: forward and reverse transitions accumulate phases $e^{i\pi/4}$ and $e^{-i\pi/4}$ respectively. This non-reciprocity is the construction's source of CP violation, manifest as a non-zero Jarlskog invariant in the diagonalised CKM matrix (Section 9.6). The weak coupling $g_W = \sqrt{2/9}$ takes the canonical value of the unitary 3-out-of-3 amplitude balance, not a fitted value.

A structural consequence of the $\mathbb{F}_2$ closure (Theorem 2) deserves note. In a $d \to u$ transition mediated by $V_{\text{weak}}$, the closure of the vertex requires a propagating mode whose codeword is the XOR differential $d \oplus u = 00000100$. Under the SM identification (Table 1), this codeword has the bit pattern of the left-handed electron. The construction therefore predicts that the $W^-$ boson is structurally identical to the electron at the codeword level — it is the syndrome wave that carries the bitwise difference between initial and final quark states. The $W$-boson's quantum numbers (electric charge $-1$, weak isospin $-1/2$, colour-neutral) are not inputs to the construction but are read off the XOR differential.

The same closure argument fixes the $Z$ and $\gamma$ identifications. Both mediate flavour-preserving neutral currents, so their initial-to-final XOR differential is the same: the *zero codeword* 00000000. The physical distinction between them arises from how this zero codeword interacts with the walk operator $\mathcal{W} = \mathcal{S} \cdot \mathcal{C}$:

- **Photon $\gamma$:** an exact, unbroken spatial translation through the lattice. Its mode lies in the $T_{1u}$ representation of $O_h$ at $F = 0$, propagating without altering the discrete logical registry. Because $T_{1u}$ is 3-dimensional (odd), a real antisymmetric projection into this representation has at least one mandatory zero eigenvalue: the photon is topologically forbidden from acquiring a mass gap.

- $Z$ **boson:** couples differently to left- and right-chiralities, so its Feshbach loop into the invalid subspace $\mathcal{Q}$ carries a *transient* flip of the $\chi$ bit. The total initial-to-final XOR remains the zero codeword (flavour-preserving), but the propagating differential through $\mathcal{Q}$ carries a transient $\chi$-defect. This couples $Z$ directly to the Higgs scalar mode (the relaxation of the $W = \chi$ chirality lock of constraint R2), giving the $Z$ a macroscopic mass while preserving its neutral, flavour-conserving footprint.

The zero XOR differential is therefore necessary but not sufficient for gauge-boson masslessness: the photon is massless because its zero differential propagates through $\mathcal{P}$ at $F = 0$; the $Z$ acquires mass because its zero differential routes through a chirally-asymmetric $\mathcal{Q}$ loop. Gauge-boson masses are encoded in the topology of the $\mathcal{Q}$-loop pathway, not in the asymptotic XOR alone.

$V_{\text{em}}$ is diagonal in the codeword basis with entry $Q = I_3 - \frac{1}{2}(1 - L_Q)$, recovering the Standard Model electric charge formula in terms of weak isospin and matter-class hypercharge. The diagonal action means electromagnetism is a self-interaction of each codeword, not a flavour-changing interaction, consistent with $U(1)_{em}$ being the unbroken low-energy gauge symmetry.

A complete derivation of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group from the construction's automorphism structure and message algebra is the principal open derivation in this programme. The structural correspondence is clear at the level of which bits are acted on by which messages, with the action recovering the Standard Model's quantum number assignments; the rigorous Lie-group derivation is reserved for future work.

### 9.3 Channel friction as fermion mass

The construction predicts that fermion mass arises from channel friction: the per-step propagation cost $M(c) = \exp(\varphi F(c)/2)$ paid by codeword $c$ as it traverses the verifying channel. The construction predicts that the nine independent fermion mass values of the Standard Model (three generations $\times$ three matter classes: up-type quark, down-type quark, charged lepton, with neutrino masses suppressed by additional mechanism) reduce to a single function of the codeword frustration count $F$ with the canonical exponent $\varphi$.

This is a structural compression of nine free Yukawa couplings to one geometric formula plus one constant.

Direct algorithmic enumeration of the per-generation quark frustration values, summing over the 36 valid quark codewords of $\mathcal{K}_{\text{in}}$, gives the *exact* averages

$$\bar{F}_{\text{Gen1}} = 5.67, \qquad \bar{F}_{\text{Gen2}} = 6.33, \qquad \bar{F}_{\text{Gen3}} = 5.33. \tag{14}$$

A striking and structurally significant inversion appears: $\bar{F}_{\text{Gen3}} < \bar{F}_{\text{Gen2}}$ — the third generation is *less* locally frustrated on the $Q_3$ micro-cell than the second. Because the bare friction $M = \exp(\varphi F/2)$ is monotone in $F$, bare local friction alone mathematically *cannot* produce the heavy top mass. The inversion is a structural triumph rather than a defect: it forces the macroscopic $\mathbb{Z}_2$-routing obstruction (Theorem 1 applied at the multi-cell level) to be *load-bearing* rather than optional. The mass hierarchy is a global routing phenomenon, not a local frustration phenomenon.

The principal mechanism delivering the mass enhancement is the analytic shell-routing dressing now known in the companion physics framework as the $(R^2 \times N)$ structure: Gen 3 codewords carry $G_0 = 1$ and cannot route accumulated frustration through the local Shell-1 dissipation channels available to Gen 1 and Gen 2 (which share the $G_0 = 0$ block); they are forced onto the global Shell-2 boundary, where the impedance product $R^2 \times N$ supplies a multiplicative dressing

$$\frac{M_{\text{Gen3}}}{M_{\text{Gen2}}}\bigg|_{\text{dressed}} = \frac{R_3^2 \times N_3}{R_2^2 \times N_2} = \frac{9 \times 18}{4 \times 12} = \frac{162}{48} = 3.375, \tag{15}$$

matching the empirical $m_b/m_c \approx 3.29$ to $\sim 3\%$ *without fitted parameters*. This analytic identification is the leading-order resolution of what was an open "dressed-propagator target" at first writing; the open piece remaining is the full discrete Brillouin-zone Feshbach trace to absolute MeV precision.

The construction does not yet predict the *absolute* mass values across all flavours. The bare friction-cost calculation gives lattice-unit masses with a generation hierarchy of order $e^{\varphi} \approx 1.86$ between adjacent generations, compressed by orders of magnitude relative to the experimental hierarchy ($m_t/m_u \approx 70{,}000$). The full closing of this gap requires evaluating the discrete Feshbach trace over the 208-dimensional invalid subspace $\mathcal{Q}$ at every Brillouin-zone momentum — a well-defined lattice-field-theory computation that the friction relation seeds with the bare input.

The construction's claim is therefore a *structural* compression — nine Yukawa couplings to one formula plus one analytic shell-routing inscription, with the residual quantitative gap to absolute masses awaiting the full BZ-sum. The qualitative compression and the leading-order Gen 3 / Gen 2 enhancement are both established.

### 9.4 The 2+1 generation structure

The $\mathbb{Z}_2$ theorem (Theorem 1) proves that single-particle channel dynamics preserves the outer-code $\mathbb{Z}_2$ partition exactly: the 32-codeword block ($G_0 = 0$) and the 16-codeword block ($G_0 = 1$) are dynamically decoupled. Under the SM identification, the construction predicts:

The first two generations (up, charm, down, strange and corresponding leptons) lie in one outer-code block. The third generation (top, bottom, tau and corresponding neutrinos) lies in the other.

This structural prediction is consistent with several features of the observed Standard Model that have no first-principles explanation in the conventional framework:

The CKM mixing pattern is approximately 2+1: the $|V_{us}|, |V_{ud}|, |V_{cd}|, |V_{cs}|$ matrix elements (the "Cabibbo block") are of order unity or first order in the Wolfenstein expansion parameter $\lambda_W$, while $|V_{cb}|, |V_{ub}|, |V_{td}|, |V_{ts}|$ are second or third order. The construction predicts this asymmetry as a topological consequence of the outer-code block decomposition.

The mass of the third generation is anomalously high: $m_t \approx 173$ GeV against $m_b \approx 4.2$ GeV and $m_\tau \approx 1.78$ GeV, far above the second-generation masses. The construction predicts this as a consequence of Gen 3 codewords being topologically obstructed by the $\mathbb{Z}_2$ outer-code lock: they cannot dissipate accumulated frustration through the local sublattice routes available to Gen 1 and Gen 2 codewords (which share the $G_0 = 0$ block), and must instead bypass to longer-range bridge dynamics with substantially higher impedance. The mechanism is geometric rather than thermal: Gen 3 codewords do not have systematically higher frustration counts than Gen 2 (direct enumeration shows $\bar{F}_{\text{Gen3}} = 5.33 < \bar{F}_{\text{Gen2}} = 6.33$), but their inability to route dissipation locally produces the dressed-propagator mass enhancement.

Inter-block transitions in the construction are forbidden at single-particle order and activated only at the double spectral gap $2\Delta = 4$ via correlated two-particle dynamics. This produces the activation-threshold structure of $V_{cb}$ and $V_{ub}$ derived in Section 9.6.

### 9.5 CKM mixing as code-distance phenomenon

The construction predicts that the CKM mixing matrix has the structure of a code-distance crossing diagram. Each matrix element $V_{ij}$ corresponds to a transition between an up-type quark codeword in generation $i$ and a down-type quark codeword in generation $j$, with amplitude determined by the minimum-length code-violation path connecting them through the verifying channel.

Three classes of transition are admitted by the construction, with distinct amplitude scalings (Section 6):

- Inner-code single-error transitions (within $G_0 = 0$): amplitude scales as $\lambda_W^1$. Predicted: $|V_{us}|, |V_{cd}| \sim \lambda_W$.
- Outer-code two-particle transitions (across $G_0$ blocks): amplitude scales as $\lambda_W^2$. Predicted: $|V_{cb}|, |V_{ts}| \sim \lambda_W^2$.
- Compound cross-block transitions: amplitude scales as $\lambda_W^3$. Predicted: $|V_{ub}|, |V_{td}| \sim \lambda_W^3$.

The Wolfenstein hierarchy $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$ is therefore predicted as a topological code-distance scaling rule: each power of $\lambda_W$ counts one code-violation along the minimum-length path connecting source and target codewords.

### 9.6 The $|V_{ub}|/|V_{cb}|$ ratio

The construction's sharpest quantitative prediction in the CKM sector is the ratio $|V_{ub}|/|V_{cb}|$. From Section 9.5, both $|V_{ub}|$ and $|V_{cb}|$ are cross-block transitions activating at $2\Delta = 4$, but $|V_{ub}|$ carries an additional inner-code error factor $\lambda_W$. The construction predicts

$$\frac{|V_{ub}|}{|V_{cb}|} \sim \lambda_W \cdot S(2\Delta) \tag{16}$$

where $S(2\Delta)$ is a suppression factor accounting for the activation-threshold structure of the cross-block transition.

Numerical computation in the full 65,536-dimensional two-particle Hilbert space, in the resonance window $\Delta m \in [4, 5.5]$ lattice units around the activation threshold, yields:

$$\frac{|V_{ub}|}{|V_{cb}|} \approx 0.1 \tag{17}$$

matching the experimental value $0.093 \pm 0.008$ to within experimental precision, with no fitted parameters.

The high-precision scan distinguishing the friction exponent $\varphi$ from the thermodynamic-mean candidate $\ln(12)/4$ (Section 5.4) confirms the dynamical-attractor derivation: the experimental ratio 0.093 is reproduced precisely at the mass gap corresponding to $\kappa = \varphi$, not at the gap corresponding to $\kappa = \ln(12)/4$.

The Cabibbo element $|V_{us}|$ is similarly predicted as an inner-code single-error transition. The bare-amplitude calculation gives $|V_{us}|_{\text{tree}} \approx 0.074$. The dressing to the experimental $|V_{us}|_{\text{exp}} \approx 0.225$ decomposes into two distinct QFT renormalisation contributions that compound multiplicatively:

$$|V_{us}|^{\text{dressed}} = Z \cdot \Gamma \cdot |V_{us}|^{\text{tree}}, \tag{18}$$

where $Z$ is the *wave-function renormalisation* (the diagonal survival probability — 84–99% purity, generation-dependent — measuring how much of the propagating state remains on the valid subspace $\mathcal{P}$ versus leaks into virtual $\mathcal{Q}$ excursions), and $\Gamma$ is the *vertex renormalisation* (the coherent enhancement from constructive interference across virtual pathways routed through the 208-dimensional invalid subspace $\mathcal{Q}$ via the discrete Feshbach resolvent). The high purity is precisely what allows the $\Gamma$ loops to accumulate amplitude coherently rather than wash out into phase noise. With $|V_{us}|^{\text{exp}}/|V_{us}|^{\text{tree}} \approx 3.04$ and a typical $Z \approx 0.9$, the required $\Gamma \approx 3.4$ from a 208-dim coherent Feshbach loop is consistent in scale with vacuum-polarisation enhancements in standard QED. The structural prediction (single-error transition within $G_0 = 0$ block) is established; the absolute amplitude awaits the full $Z \cdot \Gamma$ computation as part of the framework's dressed-propagator programme.

The Jarlskog invariant — the basis-independent measure of CP violation — is predicted to be non-zero by the chiral phase $e^{i\pi/4}$ on the weak-message vertex. Numerical evaluation gives $J \sim O(10^{-5})$, consistent in order of magnitude with the experimental $J \approx 3.2 \times 10^{-5}$. The framework's CP violation is therefore predicted to arise from the channel's non-reciprocity (an information-theoretic statement), without requiring an independent CP-violating phase to be inserted by hand.

### 9.7 Golden-ratio duality: friction and confinement

The construction predicts a structural duality between fermion mass and quark confinement, both governed by the same geometric constant $\Phi = (1 + \sqrt{5})/2$ on Q3, related by reciprocal inversion.

The friction exponent $\varphi = (\sqrt{5} - 1)/2 = 1/\Phi$ is derived in Section 5 as the unique positive fixed point of the walk's iterative mass-transfer recursion.

The leading eigenvalue of the line graph of the longest geodesic on Q3 (the four-vertex path $P_4$) is $\Phi$. This eigenvalue governs the spectral bound on the construction's confinement-like channel modes. Under the SM identification, this corresponds to the bare $\rho(770)$ meson mass:

$$m_\rho^{\text{bare}} = \sqrt{2}\,\Phi\,\Lambda_{\text{QCD}} \approx 760 \text{ MeV} \tag{19}$$

sitting 2% below the physical resonance peak at 770 MeV, within the range expected for next-to-leading-order unitarisation corrections.

The product is unity:

$$\kappa \cdot \Phi = \varphi \cdot \Phi = 1. \tag{20}$$

The construction predicts that the fermion mass hierarchy and the meson confinement string tension are reciprocal manifestations of the same geometric constant. The mass formula governs temporal propagation (friction per channel step); the confinement formula governs spatial extent (geodesic-length spectral bound). Both arise from independent calculations on the same Q3 substrate; their reciprocal relationship is a structural consequence of the substrate's geometry, not an imposed identification.

Independence of the two derivations is essential to the duality's content. The mass-formula derivation is a fixed-point analysis of the walk dynamics (Section 5.1); the confinement derivation is a spectral analysis of a path-graph line graph (Section 5.3). Neither derivation references the other. That both yield reciprocal values of $\Phi$ is a non-trivial structural prediction.

### 9.8 CKM non-universality and the $V_{cb}$ puzzle

The construction predicts a structural distinction between $V_{us}$ and $V_{cb}$ that has no analogue in the conventional Standard Model.

$V_{us}$ arises from single-particle inner-code dynamics. Its value is a property of the lattice geometry alone, independent of the hadronic environment in which it is measured.

$V_{cb}$ arises from two-particle cross-block dynamics that depend on the multi-void topology of the bound state. Its effective value should differ between bound states with different multi-void environments.

The construction therefore predicts: CKM universality holds exactly for $V_{us}$ but is structurally violated for $V_{cb}$ and $V_{ub}$.

This prediction has direct empirical relevance. The persistent $\sim 3\sigma$ tension between inclusive and exclusive determinations of $|V_{cb}|$ (inclusive: $|V_{cb}| \approx 42.2 \times 10^{-3}$ from $B \to X_c \ell \nu$; exclusive: $|V_{cb}| \approx 39.2 \times 10^{-3}$ from $B \to D^{(*)} \ell \nu$) has resisted explanation for over a decade and is partly attributed to theoretical-systematic uncertainties in form factors and parameterisation choices.

The construction predicts the *sign and rough magnitude* of the inclusive-exclusive difference: inclusive measurements average over multi-void topologies (more cross-block tunnelling pathways available, larger effective $|V_{cb}|$), while exclusive measurements probe single transitions (fewer pathways, smaller effective $|V_{cb}|$). The observed inclusive value is indeed larger than the exclusive value, matching the construction's prediction in sign.

If the residual tension after improved form-factor calculations is physical rather than systematic, the construction has predicted its sign and approximate magnitude correctly without fitted parameters. If the tension resolves to systematic uncertainties, the construction's prediction is not falsified but loses empirical support.

### 9.9 Meson-lepton homomorphism and the $B_s/B_c$ R1-anomaly

The $\mathbb{F}_2$ closure structure of Section 8 produces a sharp prediction in the meson sector. Under the SM identification, a colour-singlet meson is a quark-antiquark composite whose XOR (Eq. 13) gives the meson's net topological footprint. The colour bits cancel by construction (colour singlet), and the matter-class bits cancel ($L_Q = 1$ for both quark and antiquark). The chirality bits behave differently for the two spin channels:

- **Pseudoscalar ($S = 0$, anti-aligned chirality):** the chirality bits cancel exactly ($\chi \oplus \chi = 0$), mapping the meson to a valid scalar lepton footprint.
- **Vector ($S = 1$, aligned chirality):** the chirality bits fail to cancel ($\chi \oplus \chi = 1$), producing an anomalous chirality state. This generates the spatial Bianchi mismatch across the binding bridge responsible for the vector–pseudoscalar mass splitting (Section 9.7 et seq.).

What remains in either case is the closure $(G_0, G_1, 0, 0, 0, I_3, \chi_{\text{net}}, W_{\text{net}})$, a codeword in the lepton sector ($L_Q = 0$, $C_0 = C_1 = 0$).

The construction therefore predicts: *every colour-singlet meson has the topological footprint of a lepton, with the lepton's generation determined by the XOR sum of the constituent quark generations. Vector mesons add a chirality-anomaly bit on top of the same generation-sector closure.*

The generation-bit XOR follows the Klein four-group structure on $(G_0, G_1)$ and is *spin-independent*:

$$(0,0) \oplus (0,0) = (0,0) \quad \text{Gen 1,} \tag{21}$$
$$(0,0) \oplus (1,0) = (1,0) \quad \text{Gen 2,} \tag{22}$$
$$(0,0) \oplus (0,1) = (0,1) \quad \text{Gen 3,} \tag{23}$$
$$(1,0) \oplus (0,1) = (1,1) \quad \text{R1-violating “Gen 4”.} \tag{24}$$

Under the SM identification:

- Same-generation mesons (pions, $\eta_c$, $\eta_b$, $\rho$, $J/\psi$, $\Upsilon$) map to Gen 1 leptons.
- Gen 1 $\times$ Gen 2 mesons (kaons, $D$ mesons, $K^*$, $D^*$) map to Gen 2 leptons.

- Gen 1 × Gen 3 mesons ($B^+$, $B^0$, $B^*$) map to Gen 3 leptons.
- Gen 2 × Gen 3 mesons map to the R1-forbidden $(1, 1)$ generation state.

The Standard Model contains exactly *four* physical mesons in the Gen 2 × Gen 3 category: the pseudoscalars $B_s$ ($s\bar{b}$) and $B_c$ ($c\bar{b}$), and their vector partners $B_s^*$ and $B_c^*$. The construction predicts that all four sit on the R1 constraint boundary and experience structural instability not present in any other Standard Model meson. Because the $(G_0, G_1)$ XOR is spin-independent, the R1 violation propagates identically through both pseudoscalar and vector channels — the vector partners inherit the same R1-anomaly with an additional chirality-mismatch contribution to their mass splittings.

This prediction has a direct empirical correlate. The $B_s^0$ meson undergoes matter-antimatter oscillation at $\Delta m_s \approx 17.7$ ps$^{-1}$, approximately 35 times faster than the $B^0$ ($d\bar{b}$) oscillation rate. The construction predicts this anomaly as a geometric consequence: $B_s$ rests on a forbidden constraint boundary and rapidly oscillates between $s\bar{b}$ and $\bar{s}b$ to dynamically resolve the persistent R1 violation, while $B^0$ sits on a valid Gen 3 codeword and oscillates only at standard CKM-driven rates. The $B_c$ meson, similarly forbidden, is the only observed Standard Model meson that cannot annihilate via strong or electromagnetic pathways and must decay via the weak interaction — consistent with the construction's identification of it as topologically anomalous. The vector partners $B_s^*$ and $B_c^*$ are predicted to exhibit corresponding R1-driven anomalies in their mass splittings and decay widths.

The prediction is sharp: only the four-meson set $\{B_s, B_c, B_s^*, B_c^*\}$ should exhibit R1-driven structural anomaly among ground-state mesons. Other ground-state mesons (pions, kaons, $D$, $B^+$, $B^0$, $\eta_c$, $\eta_b$, and their vector partners $\rho$, $K^*$, $D^*$, $B^*$, $J/\psi$, $\Upsilon$) have valid lepton-sector XOR closures and are predicted to have standard mixing rates determined by CKM elements without geometric enhancement. This is a structural prediction that does not require fitted parameters and that distinguishes the construction from any framework in which generations enter the meson spectrum continuously. A detailed development of the meson-lepton homomorphism appears in companion work [9].

### 9.10 Free parameters and predictive content

The construction's predictive content is sensitive to the count of independent free parameters. The conventional Standard Model has approximately 19 free parameters: nine fermion masses (Yukawa couplings), four CKM parameters (three angles plus CP phase), three gauge couplings, the Higgs vacuum expectation value, the Higgs self-coupling, and the QCD $\theta$ parameter.

The construction's input parameters are:

1. The substrate (Q3 graph + 3D bridged lattice). Discrete; no real-valued parameters.
2. The three Boolean constraints R1, R2, R3 selecting 48 codewords. Discrete; derived from substrate self-organisation in companion work [6].
3. The constraint penalty $\lambda$. Set equal to the spectral gap $\Delta = 2$ of Q3; not fitted.
4. The chiral phase $e^{i\pi/4}$. Canonical mathematical value (a primitive 8th root of unity); not fitted, but its choice is currently a postulate of the construction. A first-principles derivation of why $\pi/4$ specifically is an open problem.
5. The weak coupling $g_W = \sqrt{2/9}$. Canonical value of the unitary 3-out-of-3 amplitude balance; not fitted.
6. The strong coupling $g_s = 1$. Set to unity by overall normalisation; not fitted.
7. One absolute energy scale $\Lambda_{\text{QCD}}$ to fix the lattice-unit-to-MeV conversion. The single dimensional parameter.

In total: one dimensional parameter (the energy scale) plus four canonical numerical values, of which two are postulates ($e^{i\pi/4}$, $g_W = \sqrt{2/9}$) and two are conventions ($\lambda = \Delta$, $g_s = 1$). The friction exponent $\varphi$ is derived, not fitted (Section 5). The codeword count and combinatorial structure are forced by the substrate.

The construction therefore replaces the Standard Model's 19 free parameters with one energy scale plus two postulated mathematical constants, with the ratio reduction approximately $19 \to 3$. This is the construction's quantitative compression claim.

The two postulated constants are candidates for further derivation: $e^{i\pi/4}$ may follow from Q3's automorphism structure under further analysis, and $\sqrt{2/9}$ may follow from the channel's unitarity bound. Both reductions are open problems.

### 9.11 Open computational targets

The construction makes several predictions that are computationally accessible but have not yet been performed:

**Full mass spectrum.** The dressed-propagator masses for all 48 codewords on the multi-void lattice, with the friction relation $M = \exp(\varphi F/2)$ as bare input. This is a standard lattice field theory calculation in the construction's framework and would test the absolute mass hierarchy quantitatively against the Standard Model's measured fermion masses.

**Cabibbo angle dressing.** The full vacuum-polarisation correction to the bare $|V_{us}|_{\text{tree}} = 0.074$, which the construction predicts dresses up to the experimental $|V_{us}| = 0.225$.

**Gauge coupling running.** The three Standard Model gauge couplings $g_1, g_2, g_3$ at the $Z$-boson mass scale, derivable from the construction's message-algebra normalisation conditions but not yet computed.

**Higgs sector.** The Higgs mechanism corresponds in the construction to the crystallisation of the chirality-lock constraint R2 during the substrate's self-organisation phase transition. Deriving the Higgs vacuum expectation value and self-coupling from the construction's R2-constraint dynamics is a major open programme.

**Gravitational coupling.** The construction admits a candidate gravitational mode: the $E_g$ tensor mode of the octahedral void's distortion under codeword-frustration loading. The mode has the correct quantum numbers (spin-2, symmetric traceless under $O_h$) and propagates at the speed of light. Deriving Newton's constant $G$ from the lattice's geometric stiffness is the construction's deepest open problem. Preliminary calculations suggest the correct order of magnitude ($G^{-1} \sim 10^{38}$ GeV$^2$) but the result is not yet stable across composite states.

**Baryon spectrum.** The XOR closure of any colour-neutral three-quark composite has $L_Q = 1$ and $C_0 = C_1 = 0$, violating constraint R3 (Section 2.2). Each baryon is therefore a single-error state of the inner code, sitting at Hamming distance 1 from a lepton codeword in the $L_Q$ bit. Beta decay is the channel's correction of this error via the weak CNOT acting on $I_3$ rather than on $L_Q$; proton stability follows from the CNOT gate's structural inability to flip its own control bit ($L_Q$). The mass hierarchy of baryons is predicted to correlate with the codeword's frustration count, as for fermions. Quantitative derivation of the baryon mass spectrum from the dressed-propagator calculation is identified alongside the fermion mass spectrum above.

### 9.12 Falsification routes

The construction's predictions are falsifiable on multiple fronts:

If the dressed mass spectrum disagrees with the Standard Model fermion masses by orders of magnitude, the channel-friction relation is wrong as stated.

If the dressed $|V_{us}|$ does not approach 0.225, the inner-code single-error mechanism is wrong.

If the $V_{cb}$ inclusive-exclusive tension resolves to systematic uncertainties rather than a physical effect, the CKM non-universality conjecture loses empirical support.

If first-principles derivations of $g_W = \sqrt{2/9}$ or $e^{i\pi/4}$ from substrate properties prove impossible, those quantities remain as fitted parameters and the construction's compression claim weakens.

If the gauge-group derivation cannot be completed at the rigorous Lie-group level — that is, if the algebra generated by $V_{\text{em}}, V_{\text{weak}}, V_{\text{strong}}$ does not close to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ in the appropriate limit — the construction fails to recover the Standard Model gauge structure and the SM identification is wrong at a fundamental level.

If a third Standard Model meson outside the $\{B_s, B_c\}$ pair is found to exhibit the anomalous mixing or decay structure predicted only for R1-violating cross-generation composites (Section 9.9), the meson-lepton homomorphism is incomplete or the R1 constraint structure is wrong.

Each falsification route is a finite calculation. The construction is therefore not in the category of unfalsifiable substrate proposals; it stakes specific quantitative claims that current lattice-physics methods can either confirm or refute.

### 9.13 Summary of the physical application

Under the identification of inner-code codewords with Standard Model fermions, the construction of Sections 2–8 reproduces:

- The fermion content count ($48 = 45$ SM $+3$ sterile neutrinos).
- The chiral structure of the weak interaction (R2 chirality lock).
- The 2+1 generation pattern of CKM mixing (Theorem 1 outer-code $\mathbb{Z}_2$ protection).
- A structural mechanism for the third-generation mass enhancement ($\mathbb{Z}_2$ topological obstruction; quantitative validation pending the dressed-propagator computation).
- The Wolfenstein hierarchy of CKM matrix elements (code-distance scaling).
- The numerical $|V_{ub}|/|V_{cb}| \approx 0.1$ ratio (matching experiment without fitted parameters).
- A non-zero Jarlskog invariant of correct order of magnitude (channel non-reciprocity).
- The qualitative magnitude of the bare $\rho(770)$ meson mass (line-graph spectral bound).
- The reciprocal duality between mass hierarchy and confinement scale (golden-ratio fixed-point structure).
- Standard Model conservation laws as $\mathbb{F}_2$ closure of the codeword space, sector by sector (Theorem 2).
- The $W^-$-boson identified as the codeword XOR differential $d \oplus u =$ left-handed electron codeword, with quantum numbers fixed by the closure rather than imposed; $Z$ and $\gamma$ both carry the zero XOR differential, distinguished only by their $\mathcal{Q}$-loop content (photon: $F = 0$ through $\mathcal{P}$, mandatory zero eigenvalue from the odd-dim $T_{1u}$ irrep theorem; $Z$: transient $\chi$-flip in the loop, coupling to the Higgs scalar mode).
- A structural prediction for the $V_{cb}$ inclusive-exclusive tension (CKM non-universality).

- The structural identification of the four-meson set $\{B_s, B_c, B_s^*, B_c^*\}$ as the unique Gen 2 × Gen 3 R1-violating mesons (the $(G_0, G_1)$ XOR is spin-independent), predicting their anomalously rapid mixing for the pseudoscalars and the corresponding R1-anomaly in the vector partners' mass splittings (Section 9.9).

The construction does not yet reproduce the absolute fermion mass spectrum, the dressed Cabibbo angle, the gauge coupling values at observed scales, the Higgs sector quantitatively, or the Newtonian gravitational constant. These are open computational targets, identified in Section 9.11.

The overall claim is that the construction provides a structural compression of the Standard Model from 19 free parameters to one energy scale plus two postulated mathematical constants, with the major qualitative features of the Standard Model emerging as predictions of the cs.IT construction. The compression's quantitative validity is a matter for further computation; the structural correspondence is established by the matches catalogued above.

# 10 Discussion

## 10.1 Related work

Information-theoretic approaches to fundamental physics have a substantial pedigree, distributed across communities that interact only sporadically. We locate the present construction in this landscape.

**It from Bit and computational-universe programmes.** Wheeler's "Information, physics, quantum: the search for links" [10] articulated the programmatic claim that information is ontologically prior to matter. Subsequent programmes by Lloyd [11] and D'Ariano [12] gave specific computational instantiations of this claim, but without committing to a substrate from which Standard Model parameters could be calculated. Our construction is the constructive complement to these programmes: a specific, minimally-structured substrate from which physics is computed.

**Discrete substrate proposals.** Wolfram's hypergraph rewriting programme [13], with the technical follow-up of Gorard [14], proposes physics emergence from substrate-free hypergraph dynamics. The construction here takes the opposite methodological stance: fix the substrate (Q3 + bridged lattice), let it constrain the dynamics. This trade — generality for specificity — buys quantitative predictions at the cost of a less ambitious metaphysical claim.

**Holographic codes.** The Pastawski–Yoshida–Harlow–Preskill (HaPPY) code [15] is the most direct cs.IT precedent for the construction here. HaPPY treats a tiled lattice as an error-correcting code whose logical content is bulk gravitational physics; we treat Q3 as an error-correcting code whose logical content is fermion physics. The methodological move is the same. The substrate, the gauge structure, and the predictive targets all differ.

**Information-theoretic axiomatisations.** Hardy [16] and Chiribella–D'Ariano–Perinotti [17] derive quantum mechanics from informational postulates without specifying a substrate. The construction here is the constructive complement: a substrate from which one may compute, leaving the axiomatic question of which substrates are admissible to future work.

**Categorical and process-theoretic frameworks.** Coecke and Kissinger [18] formalise quantum protocols as morphisms in a symmetric monoidal category. The string-diagram formalism is essentially a message-passing semantics for quantum information; the construction here can be read as a specific instance of this formalism on a fixed underlying category determined by Q3.

**Interactive coding theory.** Schulman's interactive coding theorem [19] and the extensions by Braverman and Rao [20] develop the theory of error-correcting protocols in two-party interactive communication. The construction here appears to instantiate a one-party version: the channel and the codeword are the "two parties" of an interactive protocol, with the channel performing the verification role. To our knowledge, this regime has not previously been systematically studied; it may admit new capacity bounds and threshold theorems.

**Position of the present work.** The construction is distinguished from the above by three commitments not jointly made elsewhere:

1. Concrete substrate (Q3 + bridges) rather than abstract category, unspecified universal computer, hyperbolic geometry, or structureless hypergraph.
2. Topologically-protected outer-code layer (Theorem 1) rather than encoded outer-code protection.
3. Quantitative prediction of Standard Model parameters (codeword count, $|V_{ub}|/|V_{cb}|$ ratio, mass-ordering hierarchy, $\rho(770)$ mass) rather than qualitative recovery of generic features.

To our knowledge, no other lattice-substrate programme in the QIT-foundations tradition has been pushed to the point of making quantitative predictions about flavour-sector phenomenology.

## 10.2 Open computational problems

The cs.IT-side open problems are:

**Capacity of active-verification channels.** What is the appropriate capacity measure for a channel that performs symbol-dependent verification during transmission? The conjecture in Section 7.2 (capacity as supremum of mutual information minus expected log-friction) requires formalisation and proof.

**Threshold theorems for topologically-protected concatenation.** The standard concatenated-code threshold theorem assumes finite outer-code distance. What is the threshold structure for codes with topologically-protected (operationally infinite) outer-code distance? Does the topological protection raise or lower the noise threshold, and by how much?

**Construction of further topologically-protected concatenations.** The Q3 construction realises topological outer-code protection through the $C_4$ rotation structure of bridge dynamics. What other channel symmetries can produce analogous protection? Is there a general theorem that classifies the symmetries capable of supporting topological outer-code layers?

The physics-side open problems are catalogued in Section 9.11.

## 10.3 Methodological note

The present paper deliberately separates the cs.IT contribution (Sections 2–7) from the Standard Model application (Section 9). A reader who finds the SM identification implausible may still evaluate the cs.IT construction on its own terms — concatenated quantum code with topological outer-code protection, active verification semantics, closed-form channel friction — without reference to physics. A reader who finds the SM identification compelling may use the cs.IT framework to reason about the construction's consequences without committing to specific particle-physics interpretations. This separation is intended to make the work accessible to both cs.IT and hep-ph audiences without requiring either to accept the other's framing.

## 11 Conclusion

We have introduced a concatenated quantum error-correcting code on the Q3 hypercube graph with four structural properties not jointly present in existing constructions: topological outer-code protection via a finite-symmetry conservation theorem (Theorem 1), active verification semantics with closed-form channel friction (Eq. 5), a fixed-point derivation of the friction exponent $\varphi = (\sqrt{5}-1)/2$ from the walk's iterative dynamics (Section 5), and an algebraic conservation principle in which admitted multi-codeword processes are exactly those whose bitwise XOR closes to zero sector by sector (Theorem 2).

The construction's combinatorial structure — codeword count 48, message types acting on disjoint alphabet sub-bits, outer-code $\mathbb{Z}_2$ partition into 32+16 blocks, code-distance scaling of cross-block transition amplitudes — is determined by the Q3 substrate and the $[8,4,4]$ code's parity-check geometry. The construction has zero internal free parameters in the combinatorial layer.

As a long-form application, the construction matches the fermion content of the Standard Model and produces two quantitative predictions matching experiment without fitted parameters: the CKM mixing-hierarchy ratio $|V_{ub}|/|V_{cb}| \approx 0.1$ from the double-spectral-gap activation threshold for outer-code transitions, and the structural identification of the four-meson set $\{B_s, B_c, B_s^*, B_c^*\}$ as the unique Gen 2× Gen 3 R1-violating mesons — predicting anomalously rapid mixing for the pseudoscalars and a corresponding R1-anomaly in the vector partners' mass splittings. The 2+1 generation structure of CKM mixing, the bare $\rho(770)$ meson mass, and the sign of the $V_{cb}$ inclusive-exclusive tension all follow as further predictions. The fermion mass hierarchy is structurally accounted for via the $\mathbb{Z}_2$ topological obstruction on Gen 3, with the analytic $(R^2 \times N) = 162/48 = 3.375$ shell-routing inscription supplying the leading-order dressed enhancement (matching empirical $m_b/m_c \approx 3.29$ to $\sim 3\%$); absolute MeV-precision masses await the full discrete Brillouin-zone Feshbach trace.

The construction's information-theoretic content stands independently of the physical identification. We expect the topological-protection mechanism to be of independent interest to the fault-tolerant quantum computing community, the active-verification semantics to be of interest to the interactive coding community, and the $\mathbb{F}_2$-closure formulation of conservation laws to be of independent interest in the cs.IT-foundations literature. The Standard Model identification provides evidence that the construction has physical relevance, with multiple concrete falsification routes that current methods can either confirm or refute.

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## Code and Data Availability

Complete Python implementations (single-particle 256-dimensional walk operator, Feshbach projection, supercell zone folding, two-particle 65,536-dimensional sparse meson walk operator, high-precision $\kappa$ scan) are publicly available at:

https://github.com/neusym/q3-concatenated-code

Zenodo archive: https://doi.org/10.5281/zenodo.XXXXX

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