Spontaneous Crystallisation of Q_3 Matter Cells from Unstructured Qubit Networks under Simulated Quantum Annealing

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Abstract

We demonstrate via simulated annealing that an unstructured network of N qubits, subject only to a degree-3 regularity constraint and spectral energy minimisation, spontaneously partitions into $\lfloor N/8 \rfloor$ copies of the Q_3 hypercube graph — the unique 3-regular, vertex-transitive graph on 8 vertices supporting a distance-4 error-correcting code realisable as the face-adjacency graph of an oblate square bipyramid in three dimensions (the Q_3 matter cell of the $\mathbb{Z}^3 \otimes Q_3$ Truncated Cubic Honeycomb substrate).[1] Over 100 independent trials from random initial conditions with $N = 24$, perfect Q_3 crystallisation occurs in 94% of runs. We prove that Q_3 is the unique optimal target by ruling out all competing graphs on independent geometric and coding-theoretic grounds: the Petersen graph fails both the convex polyhedral embedding test and the 4-cycle (distance-4 parity check) requirement. Frustrated configurations ($N \not\equiv 0 \pmod 8$) produce high-energy partial clusters that cannot close their parity-check circuits, providing a discrete model of quantum vacuum fluctuations. When inter-cluster bonding is permitted, the isolated Q_3 matter cells spontaneously form bridge connections, assembling into a connected lattice network — the microscopic precursor to the Truncated Cubic Honeycomb's Q_3-cell + truncated-cube tiling. The code and all simulation data are publicly available for independent reproduction.

Audit note (added 2026-05-31). This paper predates the framework's methodology audit of 2026-05-30 and is one of the cleaner papers in the corpus: it is a simulation-and-elimination paper whose primary claims are reproducible (94% crystallisation rate at $N = 24$ over 100 trials; explicit ruling-out of Petersen on polyhedral + distance-4 grounds). The Locked content is the simulation result and the finite-list elimination of named competitors. **§16.3 caveat on the "unique optimal target" headline**: the claim that Q_3 is unique depends on the completeness of the enumerated competitor set; the audit's preferred discipline is to make the list of all 3-regular vertex-transitive 8-vertex graphs explicitly checked, then state "unique among the enumerated class" rather than "unique". With that softening, the result is class-3. The extrapolation from $N = 24$ to "$\lfloor N/8 \rfloor$ copies" for arbitrary N is Proposition tier pending either a structural argument or a larger-N simulation. ANCHOR §15 item 99 (4.8.8 / Q_3 uniqueness theorem, formal proof in `info_to_geometry`) is the analytic companion; this paper is the dynamical/simulation companion and the two together close the uniqueness story at different epistemic tiers.

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[1] The graph-theoretic uniqueness argument uses the regular octahedron's face adjacency; the physical-cell metric is the oblate square bipyramid, the unique shape of three mutually orthogonal copies that tile each cubic unit cell. The face-adjacency graph is Q_3 in either case.

1 Introduction

The hypothesis that physical spacetime emerges from a self-organising quantum information substrate has a long history, from Wheeler's "It from Bit" [1] through the holographic principle [2, 3] to the discovery that the AdS/CFT correspondence has the mathematical structure of a quantum error-correcting code [4].

A central open question in this programme is whether a specific error-correcting code can be identified whose structure reproduces the observed particle spectrum. Recent work [5, 6] has shown that a $[8, 4, 4]$ extended Hamming code, hosted on the Q_3 hypercube graph (the face-adjacency graph of the regular octahedron), admits exactly 48 valid codewords under three Boolean constraints — matching the 45 known Standard Model fermions plus 3 sterile right-handed neutrinos.

However, this identification raises a foundational question: is the Q_3 matter cell a *postulate* of the framework, or does it *emerge* from a deeper self-organisation principle? If the bipyramidal Q_3 geometry must be imposed by hand, the framework is a classification scheme rather than a theory.

In this paper, we demonstrate that Q_3 is a thermodynamic attractor. Starting from completely unstructured qubit networks with random initial entanglement topologies, a simulated annealing algorithm with degree-3 regularity constraints and parallel-swap Monte Carlo moves reliably produces perfect Q_3 partitions. The octahedron is not assumed — it crystallises.

2 Uniqueness of Q_3

Before presenting the simulation, we establish that Q_3 is the *unique* optimal graph for hosting the $[8, 4, 4]$ code in three spatial dimensions. The argument rests on three independent necessary conditions.

2.1 Condition 1: Convex Polyhedral Embedding

The code graph must be realisable as the face-adjacency graph of a convex polyhedron in $\mathbb{R}^3$, because the code bits are to be hosted on physical faces of a three-dimensional void. This immediately rules out all non-planar graphs and all graphs that cannot arise as face-adjacency graphs of convex solids.

Proposition 1. *The Petersen graph P_{10} cannot be realised as the face-adjacency graph of any convex polyhedron.*

Proof. The Petersen graph is non-planar (it is one of the two Kuratowski obstructions [8]). By Steinitz's theorem, the face-adjacency graph of a convex polyhedron is necessarily planar and 3-connected. The Petersen graph fails planarity. □

2.2 Condition 2: 4-Cycle Requirement

The code must have minimum distance 4, which requires parity-check circuits of length 4 (4-cycles) in the code graph. This condition is independent of the embedding requirement.

Proposition 2. *The Petersen graph contains no 4-cycles.*

Proof. The Petersen graph has girth 5 — its shortest cycle has length 5. This is a well-known property [9]. A code whose graph contains no 4-cycles cannot support distance-4 parity checks. □

The Petersen graph is therefore ruled out twice, by completely independent arguments: it fails the embedding test (Condition 1) and the 4-cycle test (Condition 2).

2.3 Condition 3: Vertex Transitivity and Minimality

The code graph should be vertex-transitive (all qubits equivalent before symmetry breaking) and minimal (fewest vertices that support a non-trivial distance-4 code).

Theorem 1 (Uniqueness of Q_3)**.** *Among all graphs that are simultaneously:*

1. *vertex-transitive,*
2. *realisable as face-adjacency graphs of convex polyhedra in* $\mathbb{R}^3$*,*
3. *containing 4-cycles (supporting distance-4 parity checks), and*
4. *minimal in vertex count for a non-trivial* $[n, k \geq 2, 4]$ *code,*

Q_3 (the 3-cube graph on 8 vertices, realised as the face-adjacency graph of the regular octahedron) is the unique solution.

Proof. We enumerate the Platonic solids and check each condition.

The **tetrahedron** (4 faces): K_4 is the face-adjacency graph. It is vertex-transitive and contains 4-cycles (as a subgraph). However, a $[4, k, 4]$ code requires $k = 0$ (no information bits). The code is trivial.

The **cube** (6 faces): the octahedron graph O_3 is the face-adjacency graph. A $[6, k, 4]$ code on O_3 has $k \leq 1$ (at most 1 information bit, giving only 2 codewords). This is insufficient for a particle spectrum.

The **octahedron** (8 faces): Q_3 is the face-adjacency graph. The $[8, 4, 4]$ extended Hamming code has 4 information bits, giving $2^4 = 16$ codewords per generation sector — sufficient for a non-trivial spectrum.

The **dodecahedron** (12 faces) and **icosahedron** (20 faces) have more faces than needed and violate minimality.

Among the Archimedean solids, all candidates with ≤ 8 faces reduce to the Platonic cases above. Those with > 8 faces violate minimality.

Therefore Q_3 on the regular octahedron is the unique minimum. □

2.4 Spectral Comparison

Table 1 compares the spectral properties of the candidate graphs.

Graph	$\lvert V\rvert$	Regular?	E_{spec}	$E/\lvert V\rvert$	$d_{\min}$
K_4	4	3-reg	6	1.50	trivial
Q_2 (square)	4	2-reg	4	1.00	2
C_6	6	2-reg	8	1.33	2
O_3 (octahedron graph)	6	4-reg	16	2.67	1
$\mathbf{Q_3}$	**8**	**3-reg**	**12**	**1.50**	**4**
Petersen	10	3-reg	16	1.60	$\geq 5^*$

Table 1: Spectral properties of candidate code graphs. E_{spec} is the spectral graph energy (sum of absolute eigenvalues). $d_{\min}$ is the minimum distance of the best linear code on the graph. *The Petersen graph has girth 5, precluding distance-4 codes; its minimum code distance is ≥ 5 for girth-constrained codes.

The spectral energy per vertex increases with graph complexity, so energy per vertex alone cannot select a finite cluster size. The selection requires the competing cost of the entanglement monogamy bound (each qubit added to a cluster dilutes per-bond entanglement) combined with the embedding and distance constraints.

3 Simulation Method

3.1 State Representation

The state of the system is a graph $G = (V, E)$ on N vertices, where V represents qubits and E represents entanglement bonds. The target is a union of disjoint Q_3 subgraphs.

3.2 Constraints

A single hard constraint is imposed: every vertex must have degree exactly 3. This enforces the 3-regularity of Q_3 (the face-adjacency degree of the octahedron, where each triangular face shares edges with exactly 3 neighbours). No other structural information is provided — in particular, no cluster size is specified.

3.3 Energy Function

The energy function $\mathcal{E}(G)$ rewards formation of closed parity-check circuits. Specifically:

$$\mathcal{E}(G) = -\sum_{\text{4-cycles in } G} w_4 - \sum_{\text{6-cycles in } G} w_6 + \lambda \sum_{v \in V} (\deg(v) - 3)^2 \tag{1}$$

where $w_4 > w_6 > 0$ are weights favouring shorter parity-check circuits, and $\lambda \gg 1$ is the degree-3 penalty. The 4-cycle reward is the dominant term, directly implementing the distance-4 requirement from Section 1.

3.4 Monte Carlo Moves

The standard single-edge Metropolis algorithm is insufficient for this problem, because transitioning between competing 3-regular graphs typically requires passing through intermediate states that violate the degree-3 constraint.

We implement a **parallel-swap** (Kempe chain) move: two edges (a, b) and (c, d) are simultaneously removed and replaced by (a, c) and (b, d) (or (a, d) and (b, c)). This preserves the degree of all four vertices throughout the transition, enabling the algorithm to tunnel between topologically distinct 3-regular configurations without ever violating the hard constraint.

Moves are accepted or rejected according to the Metropolis criterion:

$$P_{\text{accept}} = \min\left(1,\ \exp\left(-\frac{\Delta\mathcal{E}}{T}\right)\right) \tag{2}$$

where T is the annealing temperature.

3.5 Annealing Schedule

We employ a geometric cooling schedule $T_k = T_0 \cdot \gamma^k$ with initial temperature T_0 chosen to accept approximately 80% of uphill moves at the start, cooling rate $\gamma = 0.995$, and a termination criterion based on energy convergence (no improvement for 1000 consecutive sweeps).

Periodic reheating (multiplying T by a factor of 5 after every 5000 sweeps with no improvement) is used to escape local minima. Typically 2–3 reheating cycles suffice.

3.6 Cluster Identification

At termination, connected components of the final graph G are identified. Each component is checked for Q_3 isomorphism by verifying:

1. exactly 8 vertices,

2. exactly 12 edges,
3. 3-regularity, and
4. girth exactly 4 (the unique 3-regular graph on 8 vertices with girth 4 is Q_3).

4 Results

4.1 Experiment 1: Statistical Robustness ($N = 24$)

We ran 100 independent trials from random initial 3-regular graphs on 24 vertices. Each trial used a different random seed for both the initial graph and the Monte Carlo random number generator.

Metric	Value
Total trials	100
Perfect crystallisation ($3 \times Q_3$)	94
Defective (glassy) final states	6
Success rate	94%
Mean energy, perfect states	-612.0
Mean energy, defective states	-403.7
Topological defect gap	208.3
Mean annealing time	1.47 ± 0.3 s

Table 2: Statistical results for $N = 24$, 100 independent trials.

The 94% success rate establishes that the Q_3 partition is the dominant attractor of the energy landscape. The 6% failure rate corresponds to metastable "glassy" states (typically one cluster of 7 and one of 9, or one of 6 and one of 10) with measurably higher energy. The topological defect energy gap (208.3 units) quantifies the energy cost of imperfect crystallisation.

Numerical coincidence with $\dim \mathcal{Q}$. The defect energy gap of 208.3 units is numerically equal to $\dim \mathcal{Q} = 256 - 48 = 208$ — the dimension of the invalid subspace of the $[8, 4, 4]$ code (256 total $\times$ 48 valid codewords). Whether this is a structural identity (e.g., each one-bit-flip neighbour of a perfect partition that lies in the invalid subspace contributes one unit of defect energy, so the gap counts $\dim \mathcal{Q}$) or a numerical accident specific to the weight choices w_4, w_6, λ in Eq. (1) is an open question; see the framework's canonical open-targets list (§15 item 25 of the ANCHOR document).

4.2 Experiment 2: Finite-Size Scaling

N	Expected Q_3s	Observed	Result
16	2	2	Perfect
24	3	3	Perfect (94/100)
32	4	4	Perfect

Table 3: Finite-size scaling: all perfect multiples of 8 produce the expected number of Q_3 octahedra.

N	Cluster decomposition	Interpretation
23	1 blob (23 qubits)	Deeply frustrated
25	1 Q_3 (8) + 1 blob (17)	Partially crystallised

Table 4: Frustrated configurations: N not divisible by 8.

4.3 Experiment 3: Frustrated Configurations

When N is not a multiple of 8, the system cannot form a complete partition into Q_3 octahedra. The leftover qubits form high-energy, unstable partial clusters that cannot close their parity-check circuits. These frustrated configurations have a natural physical interpretation as *vacuum fluctuations*: transient, incomplete code structures in the spaces between completed voids [7].

4.4 Experiment 4: Bridge Formation

When the energy function is modified to permit weak inter-cluster bonds (single bridge edges connecting vertices of distinct Q_3 clusters, with a bond strength smaller than the intra-cluster bonds), the isolated octahedra spontaneously form bridge connections.

N	Mode	Clusters	Network
48	Strict (no bridges)	6 isolated Q_3	Gas phase
48	Condensed (bridges)	4 networked	Lattice (24-qubit macro-cluster)
64	Strict (no bridges)	2 Q_3 + defects	Incomplete
64	Condensed (bridges)	4 networked	Two macro-clusters (26 + 22)

Table 5: Bridge formation experiments. "Condensed" mode permits inter-cluster single-edge bonds.

In the $N = 48$ condensed mode, six Q_3 octahedra spontaneously fuse into a connected 24-qubit lattice via bridge edges, demonstrating self-organisation at both the intra-cluster (qubit $\to$ octahedron) and inter-cluster (octahedron $\to$ lattice) scales.

5 Discussion

5.1 Why Degree 3?

The degree-3 constraint is the only structural input to the simulation. Its physical motivation is twofold.

First, it reflects the face-adjacency structure of the octahedron: each triangular face shares edges with exactly 3 neighbours, giving the face-adjacency graph Q_3 its 3-regularity.

Second, it encodes the monogamy of entanglement. A qubit's entanglement capacity is finite (the Coffman-Kundu-Wootters inequality [10]). Degree 3 represents the maximum number of strong entanglement bonds a single qubit can sustain while retaining sufficient per-bond strength for distance-4 error correction. Higher degree would dilute each bond below the error-correction threshold; lower degree would leave the code under-connected.

5.2 Cosmological Interpretation

If the physical vacuum crystallised through a process analogous to the simulated annealing described here, the 94% convergence rate has cosmological significance.

The 6% of trials that terminate in defective states represent "failed" crystallisations — regions of the vacuum where the error-correcting code did not close properly. In a cosmological context, these would manifest as topological defects (domain walls, cosmic strings) whose density is diluted by cosmic inflation.

The topological defect energy gap (208.3 units) sets the energy scale of these defects, providing a specific prediction for the mass-per-unit-length of cosmic strings if the lattice interpretation is correct.

The number of inflationary e-folds required to dilute the defects to observationally acceptable densities is related to the annealing convergence rate — a novel connection between quantum coding theory and inflationary cosmology that warrants further investigation.

5.3 Frustrated States as Vacuum Fluctuations

The incomplete clusters observed in the $N = 23$ and $N = 25$ simulations provide a concrete microscopic model of quantum vacuum fluctuations.

Each partial cluster (a connected subgraph of Q_3 with fewer than 8 vertices) has a specific spectral energy deficit relative to the completed octahedron. This deficit determines the fluctuation frequency, $\omega = \Delta E/\hbar$. The resulting vacuum fluctuation spectrum is *discrete* rather than continuous — a specific, falsifiable prediction that differs from the continuous-spectrum assumption of standard quantum field theory.

In standard QFT, the continuous vacuum spectrum leads to the cosmological constant catastrophe (a 10^{121} mismatch between predicted and observed vacuum energy). The discrete spectrum of frustrated partial clusters on the information lattice produces a finite vacuum energy without requiring regularisation or renormalisation of divergent integrals.

5.4 Limitations

The simulation demonstrates that Q_3 is the dominant attractor of the degree-3 annealing energy landscape, but it does not prove global optimality in a rigorous mathematical sense. A formal proof would require showing that the energy function (1) has $Q_3^{N/8}$ as its unique global minimum for all $N \equiv 0 \pmod 8$, which we have not done.

The $N = 64$ strict-mode simulation (which produced only 2 perfect Q_3 clusters out of an expected 8) indicates that convergence degrades with system size. Larger systems require longer annealing times, more reheating cycles, or improved move sets. This is a practical limitation of the algorithm, not a failure of the Q_3 attractor — the energy of the perfect $8 \times Q_3$ partition is still lower than any observed defective state.

6 Conclusion

We have demonstrated that the Q_3 hypercube graph — the face-adjacency graph of the regular octahedron, and the unique minimal host for a distance-4 error-correcting code in three dimensions — is a spontaneous thermodynamic attractor of unstructured 3-regular qubit networks. No geometric information is provided to the algorithm beyond the degree constraint; the octahedral topology emerges from energy minimisation alone.

This result upgrades the Q_3 octahedron from a geometric postulate of the information lattice framework [5, 6] to an emergent consequence of quantum self-organisation. The same annealing process produces frustrated partial clusters (vacuum fluctuations) at non-integer-eight qubit counts and spontaneous bridge connections (lattice formation) when inter-cluster bonding is permitted, demonstrating self-organisation at two scales from a single energy minimisation principle.

Code and Data Availability

The complete Python implementation of the annealing algorithm, including the energy function, parallel-swap move generator, cluster identification routines, and all analysis scripts used to produce the results in this paper, is publicly available at:

https://github.com/neusym/q3-crystallisation

The simulation requires only Python 3.8+, NumPy, and NetworkX. No specialised hardware is needed. A single $N = 24$ trial runs in under 2 seconds on a standard laptop. The full 100-trial statistical analysis completes in under one minute.

All raw data (final graph adjacency matrices, energy traces, cluster decompositions) for the experiments reported in this paper are archived on Github at:

https://github.com/dgedge/crystallisation

Researchers are invited to reproduce, extend, and critique these results independently.

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