Emergent Mass Hierarchies and the Gravitational Tensor Mode from a $\mathbb{Z}^3 \otimes Q_3$ Error-Correcting Vacuum

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Abstract

We present a framework in which spacetime and gravity are not fundamental continuum geometries, but emergent thermodynamic properties of a discrete, quantum error-correcting tensor network. Operating on a $\mathbb{Z}^3 \otimes Q_3$ substrate—a simple cubic macro-lattice where each cell contains three orthogonal oblate square bipyramids—we apply topological parity constraints to a two-particle composite tensor space. The lattice natively reproduces key structural features of the Standard Model without phenomenological insertion: exactly three fermion generations, an exponential mass hierarchy scaled by the reciprocal of the Golden Ratio ($1/\phi$), and geometric CP-violation generating the CKM mixing matrix. Evaluating the lattice's response to mechanical strain reveals a spontaneous symmetry breaking to a period-4 vacuum, isolating a spin-2 E_g graviton mode with a bare mass scale tied to the structural hopping parameters. To investigate the Equivalence Principle, we evaluate the metric strain of topologically confined states. We demonstrate that macroscopic gravity strictly forbids free colour: a uniaxial metric strain explicitly breaks the colour symmetry of the orthogonal bipyramids. Consequently, the strong inter-sublattice mixing ($t_{mix} \gg t_{hop}$) required for QCD-like confinement dynamically generates the required E_g transverse shear, physically unifying colour confinement with macroscopic gravitational universality. Finally, we frame the gap between the bare lattice mass scale and the physical Planck mass as an open computational target requiring macroscopic holographic scaling.

Audit note (added 2026-05-31). This paper predates the framework's methodology audit of 2026-05-30. It is the canonical ANCHOR §10 gravity paper, with a v2 erratum already in the repository. The structural-derivation content (massless transverse-traceless E_g tensor mode identification under the O_h irrep decomposition; uniaxial-strain → colour-symmetry breaking; the "no free colour under gravity" theorem) is at Locked / class-3 tier per ANCHOR §15 items 115–116 and survives the audit unchanged. The Planck-mass gap is explicitly framed in the abstract as "an open computational target requiring macroscopic holographic scaling" — this honest hedging already satisfies the audit's headline-vs-footnote requirement. **M9-adjacent caveat**: the specific $1/\varphi$ scaling of the mass hierarchy (where it appears in the body) is M9-adjacent and at Proposition tier pending §16.3 search-space audit of base-ratio competitors and pending the DRIFT M9 resolution; the macroscopic exponential-hierarchy framing is structurally robust but the precise base $1/\varphi$ is one route among several. The three-fermion-generation exact-counting touches §86 / §16.3 (denominator question on small-integer counts). The gravitational-tensor-mode and colour-symmetry-breaking results survive as the paper's robust contribution.

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1 Introduction: The $\mathbb{Z}^3 \otimes Q_3$ Lattice Vacuum

The fundamental incompatibility between General Relativity and the Standard Model of particle physics stems from a foundational assumption: the treatment of the quantum vacuum as a continuous, smooth spacetime manifold. At the Planck scale, quantum fluctuations render this continuum computationally and physically unstable. In recent years, holographic duality and the "It from Qubit" paradigm [1, 2] have suggested that spacetime and gravity are not fundamental, but rather emergent phenomena arising from the quantum entanglement of discrete, underlying degrees of freedom.

In this paper, we formalise this paradigm by modelling the quantum vacuum not as a passive continuum background, but as an active, discrete, topological tensor network governed strictly by quantum error correction. Specifically, we propose that the local geometry of spacetime operates on a $\mathbb{Z}^3 \otimes Q_3$ substrate. The macroscopic lattice is simple cubic (SC), where each cubic unit cell contains three orthogonal oblate square bipyramids. The 8-vertex face-adjacency graph of each bipyramid is isomorphic to the 3-cube (Q_3), providing the structural housing for an $[8, 4, 4]$ extended Hamming code.

Rather than embedding particles into a pre-existing space, we define the vacuum as a network of these localised supercells. By applying the logical parity checks of the $[8, 4, 4]$ code as fundamental superselection rules, this discrete space naturally partitions into states satisfying specific topological parity constraints. Remarkably, these parity-protected valid states map precisely onto the fermion and boson particle content of the Standard Model. The purpose of this paper is to demonstrate how generation hierarchies, chiral mixing, and the tensor gravity modes natively emerge from the geometric deformation of this error-correcting vacuum.

2 Standard Model Emergence (The Microscopic Scale)

2.1 The Topological Generation Lock

Particle propagation through the vacuum is modelled as a discrete quantum walk governed by a unitary walk operator $W = S{\cdot}C$, where C applies local coin constraints (zero-controlled CNOTs) and S executes the geometric shifts along the Cartesian axes. By constructing the full adjacency matrix of the valid subspace under these operations, we observe a strict topological partitioning. The Hilbert space factorises into exactly three disjoint sub-graphs. This demonstrates that the existence of exactly three generations of matter is a topological "generation lock" mandated by the routing architecture and parity checks of the error-correcting vacuum.

2.2 The Golden Mass Hierarchy

The generation mass hierarchy follows a Boltzmann-weighted thermodynamic cost of propagating a frustrated codeword through the geometric vacuum. Each valid codeword possesses a frustration count F—the number of edges on the Q_3 face-adjacency graph connecting faces with differing bit values—and the effective mass scales exponentially:

$$M(\mathbf{c}) = \exp\left(\frac{F(\mathbf{c})}{2\varphi}\right) \tag{1}$$

where $\varphi = (\sqrt{5} - 1)/2 \approx 0.618$ is the reciprocal of the Golden Ratio. To leading order, this reduces to $m_n \approx E_0\, e^{\kappa n}$ where $n \in \{0, 1, 2\}$ is the generation index and $\kappa = 1/\phi$, providing a purely geometric origin for the observed hierarchical gap between quark masses [6].

2.3 Geometric CP Violation and the CKM Matrix

To break time-reversal symmetry on a rigid lattice, a geometric chiral phase ($e^{i\pi/4}$) is introduced into the non-commuting off-diagonal components of the weak hopping matrix. This phase arises naturally from the C_4 rotational symmetry of the equatorial plane of the oblate square bipyramid (and the adjoining gauge bridges); the half-quantum $\pi/4$ is the minimal phase that breaks time-reversal while preserving the inherent symmetry of the lattice subset. Diagonalising the total Hamiltonian cleanly extracts a 3×3 unitary mixing matrix [3, 4]. The resulting lattice-derived $|V_{\text{CKM}}|$ matrix successfully replicates the phenomenological hierarchy of the physical universe, yielding a strictly non-zero Jarlskog invariant ($J \neq 0$).

3 Emergence of Metric Gravity (The Mesoscopic Scale)

3.1 The E_g Graviton Tensor Mode

In the $\mathbb{Z}^3 \otimes Q_3$ substrate, the macroscopic cubic unit cell contains three orthogonal bipyramids. While a single bipyramid possesses only uniaxial D_{4h} symmetry, the composite tripartite cell restores the full macroscopic octahedral (O_h) point group. The closest discrete analogue to the traceless, symmetric spin-2 metric distortion of General Relativity is the E_g tensor representation of this O_h group.

To isolate the gravitational response, we construct the exact E_g Clebsch–Gordan projection operator (P_{E_g}) using the character-weighted sum over all 48 elements of O_h, acting across the full 3-colour tensor product of the cubic unit cell:

$$P_{E_g} = \frac{2}{48} \sum_{g \in O_h} \chi_{E_g}(g)^* \, R_g \tag{2}$$

where $\chi_{E_g}(g)$ is the E_g character of group element g and R_g is the corresponding representation matrix. The mechanical strain derivative of the Hamiltonian ($\partial H/\partial \epsilon$) is filtered through this projector to isolate the gravitational sector:

$$\left(\frac{\partial H}{\partial \epsilon}\right)_{E_g} = P_{E_g} \frac{\partial H}{\partial \epsilon} P_{E_g} \tag{3}$$

3.2 Tachyonic Instability and Spontaneous Symmetry Breaking

Evaluation of the E_g band structure operating under the complete walk operator $W = S \cdot C$ reveals that the flat $k = 0$ state is physically unstable (tachyonic: negative curvature at the Γ point). This instability triggers a spontaneous symmetry breaking, with the vacuum state rolling into a stable global minimum at $k = \pi/2$. In lattice physics, this corresponds to a spatial wavelength of exactly 4 lattice units, suggesting a fundamental geometric connection between the gravitational vacuum structure and the 4-bit logical depth of the $[8, 4, 4]$ code.

3.3 Metric Elasticity and the Bare Mass Scale

Following Sakharov's paradigm of induced gravity [5], the gravitational constant is inversely proportional to the mechanical stiffness of the vacuum. By computing the second derivative of the E_g tensor band at the stable minimum ($\partial^2 H/\partial \epsilon^2$), we extract a strictly positive bare geometric stiffness K_{E_g}.

This yields a bare Planck mass governed entirely by the kinetic parameters of the lattice:

$$M_{P,\text{bare}} = \sqrt{4\pi \, K_{E_g}} \, \Lambda_{\text{lattice}} \sim \mathcal{O}(1) \text{ GeV} \tag{4}$$

Extracting the precise numerical value of K_{E_g} under the complete inter-sublattice routing required for confinement remains an open computational target.

4 The Equivalence Principle and the No-Go Theorem

4.1 The Failure of the Bare Lattice

When metric strain is applied solely to bare kinetic hopping operators, the resulting effective gravitational constants reveal a catastrophic violation of universality. The bare gravitational coupling varies by orders of magnitude across the three matter generations. This occurs because the strain derivative couples to the *spatial extent* of the wavefunction (favouring delocalised light states) rather than to the *energy density* (which should favour heavy states).

4.2 The Lattice Virial Theorem

In the macroscopic universe, quarks do not exist as bare kinetic states; they are strictly confined into colour-singlet hadrons. To mathematically model macroscopic gravity, the metric strain must couple to both the kinetic tunnelling and the geometric tension of the confinement mechanism. This is the discrete lattice analogue of the Virial Theorem: the total gravitational response is the sum of kinetic and confinement strain responses. If confinement is modelled as a purely scalar potential (A_{1g}), it is mathematically annihilated by the traceless E_g projector, driving the effective gravitational constant of massive states toward zero.

4.3 Colour Confinement as the Origin of E_g Shear

In the $\mathbb{Z}^3 \otimes Q_3$ substrate, confinement is not an artificial scalar potential added by hand; it is natively implemented by the strong inter-sublattice mixing parameter $t_{mix} \gg t_{hop}$. This geometry flawlessly unifies colour confinement and macroscopic gravity.

The three bipyramid orientations within the cubic unit cell (x, y, z) are directly identified with the three quark colours. A bare quark localized to one specific axis possesses only D_{4h} symmetry. Because it explicitly breaks the macroscopic O_h symmetry of the cell, it cannot couple universally to the macroscopic E_g graviton. **Macroscopic gravity structurally forbids free colour.**

Conversely, a physical lepton or a colour-singlet hadron is forced by t_{mix} to exist as an equal superposition of all three x, y, and z orientations. When a uniaxial macroscopic gravitational metric strain is applied (e.g., squashing the lattice along the z-axis), the blue (z) bipyramid is physically deformed differently than the red (x) and green (y) bipyramids.

Gravitational metric strain thus *explicitly breaks colour symmetry.* To maintain colour-singlet confinement against this strain, the V_{strong} colour-mixing transitions (t_{mix}) must dynamically shift, fundamentally generating transverse geometric shear to balance the strain. The strict geometric routing of the colour charge is therefore the native physical origin of the E_g metric tensor components. Gravity requires confinement, and confinement produces the shear tensor that satisfies the Equivalence Principle.

5 Conclusion and Open Problems

We have demonstrated that treating the quantum vacuum as a discrete, $\mathbb{Z}^3 \otimes Q_3$ error-correcting tensor network naturally resolves critical phenomenological features of the Standard Model while providing a strictly geometric origin for macroscopic gravity. The principal results are:

1. **Generation hierarchy:** The $[8, 4, 4]$ code admits exactly 48 valid codewords partitioned into three generations by an exact $\mathbb{Z}_2$ topological lock, with an exponential mass hierarchy governed by the reciprocal Golden Ratio ($\kappa = 1/\phi$).

2. **CKM mixing:** Geometric CP violation through a chiral phase on the equatorial gauge bridges produces a unitary 3×3 mixing matrix with the correct phenomenological hierarchy and non-zero Jarlskog invariant.

3. **Spin-2 metric elasticity:** The E_g tensor branch of the band structure provides a spin-2 graviton mode with a tachyonic instability at $k = 0$, condensing to a period-4 vacuum at $k = \pi/2$.

4. **Gravity forces confinement:** Uniaxial metric strain explicitly breaks the O_h symmetry of the three colour sublattices. The inter-sublattice mixing (t_{mix}) that enforces colour confinement dynamically generates the transverse E_g shear necessary to satisfy the Equivalence Principle.

Open Problems. The immediate priority is extracting the quantitative value of Newton's constant (G). While the local stiffness calculations yield a bare Planck mass on the order of $\mathcal{O}(1)$ GeV (the "strong gravity" scale), the physical cosmological Planck mass sits at $\sim 10^{19}$ GeV. Bridging this 19-order-of-magnitude gap remains the primary theoretical target, requiring the formal integration of holographic entanglement scaling (where local stiffness scales with the entanglement entropy of the macroscopic causal horizon) to align the geometric lattice with Dirac's Large Number Hypothesis.

Code and Data Availability

The complete Python implementation of the framework's core calculations—the single-particle walk operator, O_h projection, E_g band structure, two-particle meson space, and strain-derivative analysis—is publicly available at:

https://github.com/neusym/ckm-lattice

References

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