The Holographic Circlette: Part V
Neutrino Masses from the Lattice Koide Formula

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Abstract

We extend the circlette lattice mass formula to the neutrino sector. For charged leptons, the generalised Koide formula $m_n = \mu\,(1 + R\cos(\delta + 2\pi n/3))^2$ with $R = \sqrt{2}$ and $\delta = 2/9$ reproduces all three masses to 0.007% accuracy. The CNOT gate that generates the weak interaction uses the lepton–quark bit LQ as its control; for neutrinos ($LQ = 0$, $I_3 = 0$), the control bit is off and the gate acts as the identity. We show that this predicts $R_\nu = 1$ (a single Dirac operator path, rather than the $\sqrt{2}$ quadrature of two paths) and $\delta_\nu = 1/3 = 3/9$ (the frozen I_3 bit enlarges the effective defect from 2 to 3 of the 9 plaquette qubits). With these values, the mass-squared ratio $\Delta m^2_{21}/\Delta m^2_{31}$ matches the NuFIT 5.3 global fit to 1.6%—well within the ±2.8% experimental uncertainty on Δm^2_{21}. The predicted neutrino masses are $m_1 \approx 0.8$ meV, $m_2 \approx 8.7$ meV, $m_3 \approx 50.1$ meV, with mass sum $\Sigma m_i \approx 60$ meV (normal ordering). The neutrino Koide ratio is $Q_\nu = 1/2$ exactly, differing from the charged lepton value $Q = 2/3$ in a way that is directly traceable to the CNOT gate's inactivity. All five testable predictions—the mass ordering, the lightest mass, the mass sum, the Koide ratio, and the effective Majorana mass—are accessible to current or next-generation experiments. This is the fifth paper in a series developing the Holographic Circlette framework, but remains mathematically self-contained.

1 Introduction

In Part I of this series [1], the Standard Model fermion spectrum was derived from an 8-bit error-correcting code on a 9-qubit holographic plaquette. The circulant mass formula

$$m_n = \mu\left(1 + R\cos(\delta + 2\pi n/3)\right)^2, \qquad n = 0, 1, 2, \tag{1}$$

with the structure factor $R = \sqrt{2}$ and the twist angle $\delta = 2/9$, reproduces the charged lepton masses to remarkable precision: m_e to 0.007%, m_μ to 0.006%, and m_τ exactly (used as input to fix the overall scale μ) [2].

Both parameters have geometric origins on the lattice. The structure factor $R = \sqrt{2}$ arises from the quadrature sum of two independent Dirac operator paths on the 2D holographic surface—one per spatial dimension—that are mixed by the CNOT gate when its control bit $LQ = 1$. The twist $\delta = 2/9$ is the ratio of defect qubits ($d = 2$, the right-handed neutrino channel) to total plaquette qubits ($N = 9$). The Koide relation $Q = 2/3$ is an algebraic identity for $R = \sqrt{2}$, not a fit [6].

The natural question is whether the same formula, with geometrically determined parameters, extends to neutrino masses. The neutrinos occupy a distinguished position in the code:

they are the $I_3 = 0$, $LQ = 0$ states—the codeword 00000000 [3]. The weak CNOT gate, $I_3(t+1) = I_3(t) \oplus LQ(t)$, uses LQ as control and I_3 as target. For neutrinos, $LQ = 0$: the control bit is off. The gate acts as the identity.

This inactivity has two immediate consequences for the mass formula parameters, which we derive in Sections 2 and 3. We then confront the predictions with neutrino oscillation data in Section 4, enumerate testable consequences in Section 6, and conclude in Section 8.

2 The Structure Factor: $R_\nu = 1$

The charged lepton structure factor $R = \sqrt{2}$ originates from the Feshbach tunnelling amplitude through the ν_R defect channel. On the 2D lattice surface, the tunnelling proceeds via two independent spatial Dirac operator paths—one along each lattice axis. The CNOT gate actively mixes these two paths when $LQ = 1$, because the gate flips I_3 conditional on LQ, coupling the isospin degree of freedom to both spatial dimensions. The two paths contribute coherently in quadrature:

$$R_{\text{lep}} = \sqrt{1^2 + 1^2} = \sqrt{2}\,. \tag{2}$$

For neutrinos, $LQ = 0$. The CNOT gate does not fire. The I_3 bit is never flipped, so there is no coupling between the isospin channel and the second spatial dimension. Only a single Dirac path contributes to the tunnelling:

$$R_\nu = \sqrt{1^2} = 1\,. \tag{3}$$

This is the simplest possible prediction: $R_\nu = R_{\text{lep}}/\sqrt{2}$. The structure factor is reduced from $\sqrt{2}$ to 1 because the gate that would mix the two spatial paths is inactive.

An immediate algebraic consequence: the Koide ratio $Q \equiv \sum m_i / (\sum \sqrt{m_i})^2$ equals 2/3 exactly when $R = \sqrt{2}$, and equals 1/2 exactly when $R = 1$. The neutrino Koide ratio is therefore predicted to be

$$Q_\nu = \frac{1}{2}\,, \tag{4}$$

a clean rational value with direct geometric meaning.

3 The Twist Angle: $\delta_\nu = 1/3 = 3/9$

For charged leptons, the twist angle $\delta = 2/9$ counts the fraction of the 9 plaquette qubits occupied by the ν_R defect: $\delta_{\text{lep}} = d/N = 2/9$.

For neutrinos, the CNOT gate's inactivity has a second consequence. In the charged lepton sector ($LQ = 1$, $I_3 = 1$), the gate flips I_3, dynamically cycling it between 0 and 1. The I_3 qubit is an active degree of freedom participating in the tunnelling dynamics.

For neutrinos ($LQ = 0$, $I_3 = 0$), the gate never fires. The I_3 qubit is frozen at zero. It becomes an additional static degree of freedom—effectively enlarging the defect. Where charged leptons have $d = 2$ frozen qubits (the two ν_R channel bits), neutrinos have $d_{\text{eff}} = d + 1 = 3$: the two ν_R bits *plus* the frozen I_3 bit.

The neutrino twist angle is therefore

$$\delta_\nu = \frac{d_{\text{eff}}}{N} = \frac{3}{9} = \frac{1}{3}\,. \tag{5}$$

An equivalent reading: $\delta_\nu = \delta_{\text{lep}} + \delta_{\text{down}} = 2/9 + 1/9 = 3/9$, where $\delta_{\text{down}} = 1/9$ is the down-quark sector twist from Part I. The neutrino twist is the sum of the lepton and down-quark twists, reflecting the fact that the neutrino sits at the intersection of both sectors ($LQ = 0$, $I_3 = 0$).

4 Confrontation with Oscillation Data

4.1 The constraint structure

The mass formula (1) with $R_\nu = 1$ and $\delta_\nu = 1/3$ has one remaining free parameter: the overall scale μ_ν. Neutrino oscillation experiments measure two independent mass-squared differences [7]:

$$\Delta m^2_{21} = 7.41^{+0.21}_{-0.20} \times 10^{-5}\ \mathrm{eV}^2 \qquad \text{(solar)}\,, \tag{6}$$

$$\Delta m^2_{31} = 2.511^{+0.028}_{-0.027} \times 10^{-3}\ \mathrm{eV}^2 \qquad \text{(atmospheric, normal ordering)}\,. \tag{7}$$

Their ratio,

$$r_{\mathrm{exp}} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.02951 \pm 0.00088\,, \tag{8}$$

depends only on (R, δ) and not on μ, providing a parameter-free test of the geometric prediction.

4.2 Predicted mass-squared ratio

For $R = 1$ and $\delta = 1/3$, the three eigenvalue factors are

$$f_n = \big(1 + \cos(1/3 + 2\pi n/3)\big)^2\,, \qquad n = 0, 1, 2\,, \tag{9}$$

giving (in ascending order) $f_1 = 0.2442$, $f_2 = 0.8109$, $f_3 = 1.9450$. The predicted ratio is

$$r_{\mathrm{pred}} = \frac{f_2^2 - f_1^2}{f_3^2 - f_1^2} = 0.02997\,. \tag{10}$$

Comparing with experiment:

$$\frac{r_{\mathrm{pred}} - r_{\mathrm{exp}}}{r_{\mathrm{exp}}} = +1.6\%\,. \tag{11}$$

This is comfortably within the $\pm 3.0\%$ experimental uncertainty on r_{exp}.

4.3 Absolute masses

The overall scale μ_ν is fixed by Δm^2_{31}: $\mu_\nu^2 = \Delta m^2_{31}/(f_3^2 - f_1^2)$, giving $\mu_\nu = 1.325 \times 10^{-2}$ eV. The predicted masses (normal ordering) are:

	Predicted	
m_1	0.79	meV
m_2	8.71	meV
m_3	50.12	meV
Σm_i	59.6	meV

The mass-squared differences evaluated at these values are:

$$\Delta m^2_{21} = 7.53 \times 10^{-5}\ \mathrm{eV}^2 \qquad (\text{exp: } 7.41 \times 10^{-5},\ \text{error } 1.6\%)\,, \tag{12}$$

$$\Delta m^2_{31} = 2.511 \times 10^{-3}\ \mathrm{eV}^2 \qquad \text{(exact by construction)}\,. \tag{13}$$

4.4 Sensitivity analysis

With $R = 1$ fixed, the exact twist angle that reproduces r_{exp} is $\delta_{\mathrm{exact}} = 0.33059$, differing from $1/3 = 0.33333$ by 0.82%. This is well within the propagated uncertainty from Δm^2_{21}, confirming that the geometric value $\delta = 1/3$ is consistent with current data.

4.5 Geometric candidate comparison

To confirm that $(R = 1, \delta = 1/3)$ is not an *ad hoc* choice but the best geometric candidate, we systematically tested all lattice-motivated (R, δ) pairs against the mass-squared ratio constraint $r = r_{\text{exp}}$. Table 1 ranks the top candidates by their deviation from experiment.

Table 1: Geometric (R, δ) candidates ranked by agreement with the observed mass-squared ratio $r_{\text{exp}} = 0.02951$. The error column gives the percentage deviation of the predicted r from experiment. Only candidates with sub-30% error are shown.

R	δ	r_{pred}	Error	Physical motivation
1	1/3	0.02997	1.6%	CNOT off: single path, frozen I_3 defect
$1/\sqrt{3}$	1/9	0.02721	7.8%	$1/\sqrt{N_c}$ suppression, down-sector twist
$\sqrt{2}$	4/9	0.02313	21.6%	Same as leptons, double twist
$\sqrt{2}/3$	1/9	0.03725	26.2%	R_{lep}/N_c, down-sector twist

The $(R = 1, \delta = 1/3)$ pair is the clear winner. The next-best candidate—$R = 1/\sqrt{3}$ with the down-quark twist—deviates by 7.8%, outside the 1σ experimental band. The charged-lepton values $(R = \sqrt{2}, \delta = 2/9)$ give $r = 0.0035$, off by a factor of 8, confirming that the neutrino sector requires different parameters, as expected from the CNOT gate's inactivity.

5 Comparison of Sectors

With the neutrino result, the mass formula parameters for all four fermion sectors can be tabulated:

Table 2: Mass formula parameters across fermion sectors. All values of δ are ratios of small integers over 9 (or 27 for up quarks, i.e. $9 \times N_c$). The Koide ratio Q is an algebraic consequence of R. Quark values are from Part I [1]; the neutrino values are derived in this paper.

Sector	R	δ	Koide Q	Tier	CNOT status
Charged leptons	$\sqrt{2}$	2/9	2/3	1 (exact)	Active ($LQ = 1$)
Neutrinos	1	$1/3 = 3/9$	1/2	1b (this paper)	Inactive ($LQ = 0$)
Down quarks	~ 1.55	1/9	—	2 (fitted)	Active, colour-dressed
Up quarks	$\sim \sqrt{3}$	2/27	—	2 (fitted)	Active, colour-dressed

The pattern in δ is systematic: $\delta = d/N$ where d counts the effective number of frozen defect qubits visible to each sector. For charged leptons, $d = 2$ (the ν_R channel). For neutrinos, $d = 3$ (the ν_R channel plus the frozen I_3 bit). For down quarks, $d = 1$ (colour dilution by N_c). For up quarks, $d = 2$ with $N \to N \times N_c = 27$ (colour dilution of both the defect and the plaquette).

The pattern in R is equally systematic: $R = \sqrt{2}$ when the CNOT mixes two spatial paths; $R = 1$ when it does not; $R \sim \sqrt{3}$ when three colour paths contribute (with NLO gluon dressing shifting the effective value).

6 Testable Predictions

The $(R = 1, \delta = 1/3)$ neutrino mass formula generates five predictions accessible to current or planned experiments:

1. **Normal mass ordering.** The formula gives $m_1 < m_2 < m_3$ with a strong hierarchy between the lightest and heaviest states ($m_3/m_1 \approx 63$). The JUNO experiment [8] will determine the mass ordering at 3σ within its first six years of operation.
2. **Lightest neutrino mass: $m_1 \approx 0.8$ meV.** This places the spectrum in the "normal hierarchy" regime with a near-massless lightest state. Future long-baseline oscillation experiments and cosmological surveys will constrain m_1; the predicted value is far below the current KATRIN upper bound of 450 meV [9] but within reach of next-generation direct mass measurement concepts.
3. **Mass sum: $\Sigma m_i \approx 60$ meV.** The Planck satellite combined with baryon acoustic oscillation data gives the current upper bound $\Sigma m_i < 120$ meV (95% CL) [10]. The CMB-S4 experiment combined with DESI is projected to reach a sensitivity of ~ 30 meV, which will either confirm or rule out the prediction.
4. **Neutrino Koide ratio: $Q_\nu = 1/2$.** Once the absolute neutrino mass scale is measured (by any combination of oscillation, beta decay, and cosmological probes), the Koide ratio can be computed directly. The prediction $Q_\nu = 1/2$ differs from the charged lepton value $Q = 2/3$ and from the "democratic" value $Q = 1/3$; it is a sharp discriminator between mass models.
5. **Effective Majorana mass: $|m_{ee}| \approx 3.7$ meV.** Using approximate PMNS matrix elements ($|U_{e1}|^2 \approx 0.67$, $|U_{e2}|^2 \approx 0.30$, $|U_{e3}|^2 \approx 0.022$), the effective mass governing neutrinoless double-beta decay is

$$|m_{ee}| = \Big|\, |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3 \,\Big| \approx 3.7\ \text{meV}\,. \tag{14}$$

   This is below the current best limits ($|m_{ee}| < 36$–156 meV, KamLAND-Zen [11]) but within the projected sensitivity of tonne-scale $0\nu\beta\beta$ experiments such as LEGEND-1000 and nEXO ($\sigma \sim 10$–20 meV).

7 Why Neutrino Masses Are Tiny

The mass formula (1) separates the neutrino mass puzzle into two independent questions: *why is the overall scale μ_ν so small?* and *why is the intra-generational hierarchy (m_3/m_1) less extreme than for charged leptons?*

The second question is answered by the structure factor. For charged leptons, $R = \sqrt{2} \approx 1.414$, which places the electron near a spectral node: $(1+\sqrt{2}\cos\theta_e) \approx 0.040$, giving a mass ratio $m_\tau/m_e \approx 3477$. For neutrinos, $R = 1$ keeps all three eigenvalues further from zero, compressing the hierarchy: $m_3/m_1 \approx 63$. The intra-generational hierarchy is less extreme because the CNOT gate's inactivity prevents the quadrature enhancement that pushes the electron close to the spectral node.

The first question—why $\mu_\nu \approx 1.3 \times 10^{-2}$ eV compared with $\mu_{\text{lep}} \approx 314$ MeV—requires a separate mechanism. The ratio $\mu_\nu/\mu_{\text{lep}} \sim 4\times 10^{-11}$ is characteristic of seesaw-scale suppression. In the circlette framework, the ν_R state is excluded from the minimal code by constraint R4. Mass generation for neutrinos proceeds through tunnelling into this excluded state, and the tunnelling amplitude is exponentially suppressed by the code distance separating the valid codeword space from the ν_R pseudocodeword. Deriving the precise suppression factor from the lattice's fault-tolerance parameters remains an open problem.

8 Conclusion

The circlette lattice mass formula extends naturally to the neutrino sector with both parameters—the structure factor $R = 1$ and the twist angle $\delta = 1/3$—determined by the CNOT gate's inactivity on $I_3 = 0$, $LQ = 0$ states. No free parameters are introduced beyond the overall mass scale μ_ν. The predicted mass-squared ratio matches oscillation data to 1.6%, the mass sum satisfies cosmological bounds, and the neutrino Koide ratio $Q = 1/2$ provides a sharp, testable prediction.

With this result, the circlette framework now derives or constrains masses in all four fermion sectors—charged leptons ($R = \sqrt{2}$, $\delta = 2/9$), neutrinos ($R = 1$, $\delta = 1/3$), down quarks ($R \approx 1.55$, $\delta = 1/9$), and up quarks ($R \approx \sqrt{3}$, $\delta = 2/27$)—from a single formula whose two parameters have transparent geometric meaning on the lattice.

The programme continues: derivation of the PMNS neutrino mixing matrix from the quantum walk operator (applying the Part IV machinery to the $LQ = 0$ sector), computation of the seesaw suppression factor μ_ν/μ_{lep} from the lattice's fault-tolerance threshold, and systematic verification against upcoming results from JUNO, DESI, and tonne-scale $0\nu\beta\beta$ experiments.

The Python scripts used for the numerical analysis are available at `https://github.com/dgedge/circlette-neutrino`.

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