The Holographic Circlette: Part VIII Hadron Topology, the Meson-Lepton Homomorphism, and the Geometric Origin of B_s Mixing

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Abstract

We extend the 8-bit Holographic Circlette framework to hadronic bound states by evaluating mesons as topological XOR composites of their constituent quark and antiquark codewords. Because physical mesons are rigid $SU(3)$ colour singlets, $\mathbb{F}_2$ Boolean logic algebraically forces their internal colour bits to identically cancel, mapping the entire pseudoscalar meson nonet strictly into the colourless lepton subspace. This reveals an exact topological homomorphism between strongly interacting meson flavour states and fundamental weakly interacting leptons. Strikingly, the composite of the π^+ pion mathematically evaluates to the absolute null codeword ('00000000'), providing a rigorous geometric analogue for the pion as the pseudo-Goldstone excitation of the QCD vacuum. Furthermore, evaluating cross-generational bound states reveals that the B_s and B_c mesons uniquely map to the $(G_0 = 1, G_1 = 1)$ generation state, violently violating the framework's primary R1 topological constraint. We propose that this severe discrete structural anomaly natively explains why $B_s^0 - \bar{B}_s^0$ mixing occurs at phenomenologically extreme frequencies compared to all other Standard Model mesons.

1 Introduction

In the Holographic Circlette framework [1], fundamental Standard Model fermions are encoded as 8-bit topological states on a discrete Boolean hypercube, governed by four rigid constraints (R1–R4). Having successfully derived the fundamental fermion spectrum, continuous gauge anomaly cancellations [5], and the exact structural origins of both the CKM and PMNS mixing matrices [3, 4] strictly from topological operators acting on these individual discrete states, the natural progression is to examine macroscopic hadronic bound states.

In Quantum Chromodynamics (QCD), a meson is a bound state of a quark and an antiquark. Topologically, a composite state of two fermions on a Boolean hypercube can be mathematically evaluated via the bitwise XOR ($\oplus$) of their constituent 8-bit strings. This operation captures the conserved topological quantum numbers of the composite system in $\mathbb{F}_2$ algebra.

In this paper, we evaluate the XOR composites of the pseudoscalar meson nonets. We discover an exact, mathematically rigid structural homomorphism between the complex multiplet structure of mesons and the fundamental lepton generations, yielding discrete geometric derivations for the Goldstone boson vacuum and the phenomenological anomaly of rapid B_s meson mixing.

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2 The Boolean Antiquark and the Colour Singlet

The 8-bit fermion state is encoded as $(G_0, G_1, LQ, C_0, C_1, I_3, \chi, W)$. To evaluate the topological composite of a meson, we must formally define the topological representation of the antiquark ($\bar{q}$) within the discrete code.

2.1 The Colour Singlet Requirement

By definition, a physical macroscopic meson must be a strict $SU(3)$ colour singlet (it carries zero net colour charge). In continuous QCD, this is achieved via a coherent superposition $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$. In the discrete $\mathbb{F}_2$ Boolean logic of the hypercube, the physical requirement of "net-zero colour" strictly translates to the exact annihilation of the colour bits under the XOR operation:

$$C_{0(q)} \oplus C_{0(\bar{q})} = 0, \qquad C_{1(q)} \oplus C_{1(\bar{q})} = 0 \tag{1}$$

Because $x \oplus y = 0 \iff x = y$ in $\mathbb{F}_2$, the requirement of forming a colour singlet mandates that the antiquark possesses the *identical* discrete Boolean colour coordinates as its partner quark.

2.2 Isospin Conjugation

Furthermore, classical charge conjugation strictly reverses the physical weak isospin of the state. Therefore, an up-type antiquark ($\bar{u}, \bar{c}, \bar{t}$) inherently carries $I_3 = 1$, and a down-type antiquark ($\bar{d}, \bar{s}, \bar{b}$) inherently carries $I_3 = 0$. Both constituents retain $LQ = 1$ (they remain quark-type fundamental constituents).

3 The Meson-Lepton Homomorphism

If we evaluate the generic XOR composite of a colour-singlet meson, the result is topologically profound:

$$LQ_{\text{comp}} = 1 \oplus 1 = \mathbf{0} \tag{2}$$

$$C_{\text{comp}} = C_q \oplus C_{\bar{q}} = (\mathbf{0}, \mathbf{0}) \tag{3}$$

$$\chi_{\text{comp}} = 0 \oplus 0 = \mathbf{0} \quad \text{(for left-left pseudoscalar coupling)} \tag{4}$$

Because the composite strictly evaluates to $LQ = 0$ and $C = (0, 0)$, **the Boolean lattice mathematically forces every valid colour-singlet meson to project flawlessly onto the topological footprint of a left-handed lepton.**

The specific lepton target is governed entirely by the generation bits, which naturally form a closed Klein Four-Group (V_4) operation under XOR:

$$(0,0) \oplus (0,0) = (0,0) \quad \text{(Gen 1)} \tag{5}$$

$$(0,0) \oplus (1,0) = (1,0) \quad \text{(Gen 2)} \tag{6}$$

$$(0,0) \oplus (0,1) = (0,1) \quad \text{(Gen 3)} \tag{7}$$

$$(1,0) \oplus (0,1) = (1,1) \quad \text{(Forbidden Gen 4)} \tag{8}$$

This establishes an exact topological homomorphism categorising all Standard Model pseudoscalar mesons strictly by their resultant leptonic boundary:

- **Maps to Gen 1 Lepton (e, ν_e):** All same-generation bound states (Pions, η, η_c, η_b).
- **Maps to Gen 2 Lepton (μ, ν_μ):** All Gen 1 $\times$ Gen 2 states (Kaons, D mesons).
- **Maps to Gen 3 Lepton (τ, ν_τ):** All Gen 1 $\times$ Gen 3 states (B^+, B^0).

The vast, complex combinatorial space of QCD meson nonets is mathematically projected by the hypercube directly down to the simple, exact basis of the three fundamental lepton generations.

4 The Pion as the Topological Vacuum

The physical consequences of this homomorphism are immediately apparent when evaluating the lightest meson in the Standard Model: the charged pion ($\pi^+ = u\bar{d}$).

- The u quark (Generation 1): $G = (0, 0)$, $I_3 = 0$.
- The $\bar{d}$ antiquark (Generation 1): The native d quark possesses $I_3 = 1$, meaning its charge conjugate $\bar{d}$ possesses $I_3 = 0$.

Evaluating the full bitwise XOR:

$$\pi^+_{\text{comp}} = (0, 0, 1, C_0, C_1, 0, 0, 0) \oplus (0, 0, 1, C_0, C_1, 0, 0, 0) = \mathbf{00000000} \tag{9}$$

The composite topology of the π^+ meson mathematically evaluates to the exact **null codeword** of the 8-bit hypercube (the geometric vacuum state, corresponding to the ν_{eL}).

In continuum Quantum Field Theory, the anomalous lightness of the pion is famously explained by identifying it as a pseudo-Nambu-Goldstone boson—an explicit macroscopic excitation of the chiral QCD vacuum itself. The discrete hypercube reproduces this profound truth geometrically: the flavour topology of the fundamental mediator of the strong nuclear force is literally indistinguishable from the absolute structural ground state of the graph.

Similarly, evaluating the neutral π^0 (approximated by the $u\bar{u}$ component) effectively evaluates $u \oplus \bar{u}$, yielding an identical null state but with $I_3 = 0 \oplus 1 = 1$. The π^0 physically evaluates to the electron state (e_L). The lattice perfectly conserves and mirrors the exact $SU(2)$ isospin multiplet structure of the pion family.

5 The Forbidden 4th Generation and the B_s Mixing Anomaly

Evaluating the heavy meson states reveals a striking structural anomaly driven by the final permutation of the Klein four-group.

When evaluating mesons comprised of Generation 2 and Generation 3 combinations, the generation bits sum to $(1, 0) \oplus (0, 1) = (\mathbf{1}, \mathbf{1})$. This specific geometric coordinate natively violates Rule 1 (R1) of the 8-bit error-correcting code ($G_0 \cdot G_1 \neq 1$), which fundamentally establishes the three-generation limit of the physical Standard Model [1].

There are only two ground-state pseudoscalar mesons in the Standard Model that possess this specific physical quark combination: the B_s ($s\bar{b}$) and the B_c ($c\bar{b}$). The Boolean lattice dictates that these states uniquely evaluate to the structurally forbidden "Fourth Generation." Consequently, they cannot exist as geometrically stable, static footprints within the valid codeword subspace.

This discrete geometric constraint violation identically mirrors one of the most extreme phenomenological observations in particle physics:

1. **Rapid B_s Mixing:** The neutral B_s^0 meson undergoes matter-antimatter oscillation at a blindingly fast frequency ($\Delta m_s \approx 17.7$ ps^{-1})—roughly 35 times faster than standard B^0 ($d\bar{b}$) mixing [6]. The discrete lattice mathematically isolates this anomaly: while the B^0 sits statically on a valid Gen 3 topological coordinate, the B_s^0 rests precisely on a forbidden constraint boundary. It is under massive geometric stress, forcing it to rapidly and violently oscillate between its $s\bar{b}$ and $\bar{s}b$ states to dynamically resolve the persistent R1 violation.

2. B_c **Weak Decay:** The B_c^+ meson is mathematically unique as the only observed Standard Model meson composed of two differing heavy quarks. It fundamentally cannot annihilate via Strong or Electromagnetic pathways; it must decay via the Weak interaction. It is mathematically anomalous in continuous QCD, perfectly matching its mathematically anomalous status in the discrete code.

6 Conclusion

The application of discrete Boolean $\mathbb{F}_2$ logic to hadronic bound states does not yield a simple continuous kinematic mass formula, but rather exposes the underlying **Algebraic Topology of the Strong Force**.

The discrete framework provides a strict mathematical formulation for the pion as a Goldstone-like excitation of the absolute vacuum ('00000000'). It establishes a rigorous homomorphism mapping conserved flavour quantum numbers from the macroscopic hadronic spectrum directly onto the fundamental leptonic generations via the Klein four-group. Finally, it strictly isolates the B_s and B_c mesons as the sole topological R1-constraint violators, providing a profound geometric mechanism for anomalously rapid particle-antiparticle mixing phenomena.

These results prove that the Holographic Circlette structure extends deeply into the composite hadron sector, defining discrete topological conservation laws that accurately reflect complex macroscopic QCD phenomenology.

References

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A Computational Appendix

The topological meson-lepton homomorphism and the structural violation of constraint R1 by the cross-generational B_s and B_c mesons can be verified programmatically using the following precise $\mathbb{F}_2$ logic sequence.

```
def xor_states(s1, s2):
    return tuple(a ^ b for a, b in zip(s1, s2))

def get_quark_state(gen, flavour):
    gen_bits = {1: (0,0), 2: (1,0), 3: (0,1)}[gen]
    I3 = 0 if flavour == 'u' else 1
    # Colour natively set to 'red' (0,1), Chirality Left (0)
    return (gen_bits[0], gen_bits[1], 1, 0, 1, I3, 0, 0)

def get_antiquark_state(gen, flavour):
    gen_bits = {1: (0,0), 2: (1,0), 3: (0,1)}[gen]
    # Antiquark flips I3. Colour bits must identically match
    # to guarantee F2 Colour Singlet XOR cancellation.
    I3 = 1 if flavour == 'u' else 0
    return (gen_bits[0], gen_bits[1], 1, 0, 1, I3, 0, 0)

def evaluate_hadrons():
    pseudoscalar_mesons = [
        ("pi+",   1, 'u', 1, 'd'),
        ("pi0",   1, 'u', 1, 'u'),
        ("K+",    1, 'u', 2, 'd'),
        ("D0",    2, 'u', 1, 'u'),
        ("B+",    1, 'u', 3, 'd'),
        ("B0",    1, 'd', 3, 'd'),
        ("eta_c", 2, 'u', 2, 'u'),
        ("Bs",    2, 'd', 3, 'd'),
        ("Bc+",   2, 'u', 3, 'd')
    ]

    lepton_names = {
        ((0,0), 0): "nu_e (Vacuum)", ((0,0), 1): "e",
        ((1,0), 0): "nu_mu",         ((1,0), 1): "mu",
        ((0,1), 0): "nu_tau",        ((0,1), 1): "tau",
    }

    print(f"{'Meson':<8} {'q':<12} {'qbar':<14} {'Composite':<14} {'Mapped Lepton'}")
    print("-" * 75)

    for name, qg, qf, qbg, qbf in pseudoscalar_mesons:
        q = get_quark_state(qg, qf)
        qbar = get_antiquark_state(qbg, qbf)

        comp = xor_states(q, qbar)
        G0, G1, I3 = comp[0], comp[1], comp[5]

        gen_xor = (G0, G1)
```

```
        lepton = lepton_names.get((gen_xor, I3), "UNKNOWN")

        marker = " <-- STRUCTURAL R1 ANOMALY" if (G0==1 and G1==1) else ""
        comp_str = ''.join(map(str, comp))

        print(f"{name:<8} ({qg},{qf})       ({qbg},{qbf}bar)       "
              f"{comp_str:<14} {lepton}{marker}")

if __name__ == "__main__":
    evaluate_hadrons()
```