The Holographic Circlette: Part XVII Topological Thawing and the Exact Geometric Derivation of the Dark Energy Trajectory w_a

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Abstract

Recent cosmological observations from the Dark Energy Spectroscopic Instrument (DESI) strongly indicate that Dark Energy is not a static cosmological constant ($w = -1$), but rather an evolving dynamical fluid. In prior work, we derived a present-day equation of state $w_0 = -3/4$ via exact topological code-counting of the Holographic Circlette vacuum. In this paper, we derive the dynamic trajectory parameter w_a without continuous free parameters. By treating the vacuum as an emergent computational tensor network (the "Circlette Soup"), the absolute origin of the universe ($a \to 0$) must correspond to a perfectly symmetric, pre-error-corrected Boolean state. At this infinite-density limit, the framework's generation constraint ($G_0 \cdot G_1 \neq 1$) cannot be structurally enforced, rendering all four boolean generation states mathematically degenerate and enforcing a strict initial boundary condition of $w(0) = -1$. Applying the standard CPL parameterisation $w(a) = w_0 + w_a(1-a)$, this topological boundary strictly mandates $w_a = -1/4$. This provides a first-principles derivation of a "thawing" dark energy trajectory $w(a) = -1 + a/4$. Strikingly, this exact geometric sequence guarantees a finite, non-singular dark energy density at the Big Bang ($\approx 2.117\rho_0$) and predicts that at scale factor $a = 4$, the vacuum will decay into a pressureless state ($w = 0$), natively averting the infinite de Sitter heat death of the cosmos.

1 Introduction

For over two decades, the standard ΛCDM model of cosmology has assumed that Dark Energy is an immutable property of empty space, perfectly described by a cosmological constant with an equation of state $w = p/\rho = -1$. However, the 2024 data release from the Dark Energy Spectroscopic Instrument (DESI) provided compelling multi-sigma evidence that dark energy is dynamic and weakening over time [1].

Dynamic dark energy is classically tracked using the Chevallier-Polarski-Linder (CPL) parameterisation:

$$w(a) = w_0 + w_a(1-a) \tag{1}$$

where a is the cosmological scale factor, w_0 is the current equation of state, and w_a dictates the evolutionary trajectory. Current global fits strongly prefer $w_0 > -1$ and $w_a < 0$, but standard continuum field theories (such as quintessence) offer no mechanism to strictly fix these values, treating them as arbitrary phenomenological curve-fitting parameters subject to tuned scalar potentials.

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In the Holographic Circlette framework, the vacuum is not an empty, continuous void. It is an emergent 3D spacetime generated by the algorithmic execution of quantum error-correcting codes across a pre-geometric network of 8-bit circlettes [2, 3]. In Part XVI, we derived the current equation of state $w_0 = -3/4$ strictly from the algebraic exclusion of the forbidden $(1, 1)$ boolean generation state from the active matter spectrum, causing it to act as a topological defect exerting negative informational pressure on the macroscopic lattice.

In this paper, we extend this algorithmic geometry to the temporal evolution of the universe, deriving the exact fractional value of the dynamic trajectory parameter w_a.

2 The Algorithmic Thawing of the Vacuum

In the discrete "Circlette Soup" ontology, spatial volume is directly proportional to the total number of instantiated error-corrected null-codewords in the topological tensor network. Therefore, the physical expansion of the universe (increasing a) is fundamentally an algorithmic, code-copying process.

To determine the dynamic evolution of the vacuum's pressure, we must establish the boundary condition of the network at the Big Bang ($a \to 0$).

At the absolute origin of the quantum walk, the tensor network consisted of a highly dense, unexpanded symmetric state. Because the spatial lattice lacked the macroscopic diameter necessary to execute long-range fault-tolerant error correction, the foundational Boolean constraint R1 ($G_0 \cdot G_1 \neq 1$) could not be physically enforced. Consequently, the 4th generation $(1, 1)$ was not yet structurally isolated; all four generation coordinate slots were mathematically degenerate and equally active.

Because 4 out of 4 states contributed to the pre-geometric vacuum pressure at the singularity, the initial equation of state of the discrete network operated as a perfect, exact cosmological constant:

$$w(0) = -\frac{4}{4} = -1 \tag{2}$$

In cosmological terms, this structurally defines the framework as a **Thawing Vacuum Model**. The universe begins with its dark energy perfectly frozen at $w = -1$, driving initial exponential inflation. As the scale factor a increases and the network expands, macroscopic error-correction is achieved, freezing out the $(1, 1)$ state. This topological symmetry breaking causes the negative pressure of the vacuum to "thaw" and gradually relax toward its fractional geometric limit of $-3/4$.

3 Exact Derivation of w_a

We apply the strict topological boundary condition $w(0) = -1$ directly to the CPL parameterisation:

$$w(a) = w_0 + w_a(1 - a) \tag{3}$$

$$w(0) = w_0 + w_a(1 - 0) \tag{4}$$

$$-1 = w_0 + w_a \tag{5}$$

This reveals a profound geometric constraint: the sum of the current equation of state and the trajectory parameter must perfectly evaluate to the negative identity of the original unbroken vacuum.

Substituting the geometrically derived current-era value $w_0 = -3/4$:

$$-1 = -\frac{3}{4} + w_a \tag{6}$$

$$w_a = -1 + \frac{3}{4} \tag{7}$$

$$w_a = -\mathbf{\frac{1}{4}} = -0.25 \tag{8}$$

The trajectory parameter is geometrically fixed to exactly $w_a = -0.25$. This theoretical pairing ($w_0 = -0.75, w_a = -0.25$) sits flawlessly within the preferred phenomenological bounds of recent observations, achieved with absolute algebraic rigidity and zero free continuous parameters.

4 Finite Density and the Averted Heat Death

By substituting the exact fractions back into the dynamic trajectory, we reveal the complete analytical function for Dark Energy in the Holographic Circlette universe:

$$w(a) = -\frac{3}{4} - \frac{1}{4}(1-a) = \mathbf{-1 + \frac{a}{4}} \tag{9}$$

This exact linear equation resolves two of the most catastrophic infinities in continuous cosmology.

First, evaluating the physical density of dark energy ρ_{DE} backward in time typically yields mathematically destructive singularities in dynamic fluid models. However, integrating the Circlette trajectory yields a perfectly bounded exponential decay:

$$\rho_{DE}(a) = \rho_0 \exp\left(3\int_a^1 \frac{1+w(a')}{a'}da'\right) = \rho_0 \exp\left(\frac{3}{4}(1-a)\right) \tag{10}$$

Evaluating at the Big Bang ($a = 0$), the initial vacuum density was exactly:

$$\rho_{DE}(0) = e^{3/4}\rho_0 \approx \mathbf{2.117\rho_0} \tag{11}$$

The dark energy density is strictly finite and non-singular at the moment of creation.

Secondly, in standard ΛCDM ($w = -1$), the universe accelerates exponentially forever, terminating in the "Heat Death" of infinite, empty de Sitter space. This presents severe philosophical paradoxes, including the fundamental breakdown of string-theoretic S-matrices.

However, in the discrete Circlette trajectory, $w(a)$ is steadily rising. The universe only undergoes accelerated expansion ($\ddot{a} > 0$) while $w < -1/3$. We can calculate exactly when cosmic acceleration will naturally switch off:

$$-\frac{1}{3} = -1 + \frac{a_{\text{crit}}}{4} \tag{12}$$

$$a_{\text{crit}} = \frac{8}{3} \approx \mathbf{2.66} \tag{13}$$

Furthermore, when the universe expands to exactly four times its present radius ($a = 4$), $w(4) = 0$. The algorithmic overhead of copying the discrete tensor network fully balances the intrinsic vacuum energy, and Dark Energy gracefully decays into a pressureless, matter-like state (topological dust). The Holographic Circlette uniquely derives a universe completely safe from infinite eternal expansion, natively permitting a stable, finite cosmological future.

4.1 Algorithmic Work vs. Volumetric State-Counting

A critical statistical refinement must be addressed regarding the derivation of the macroscopic vacuum pressure $w_0 = -3/4$. A naive thermodynamic approach might attempt to weight the vacuum energy contribution of each constraint rule by the absolute combinatorial volume of the Hilbert space it excludes from the 256-state hypercube.

If we evaluate the strict exclusion counts per generation: constraint R1 explicitly forbids 64 states (the 4th generation); R2 excludes 80 states (chirality mismatch); R3 excludes 26 states (coloured leptons/colourless quarks); and R4 strictly isolates only 3 states (the right-handed neutrinos). Weighting the equation of state by these exclusion subsets would yield a macroscopic average overwhelmingly dominated by the structural bulk constraints:

$$w_{0(\text{naive})} = \frac{64(-1) + 80(-1) + 26(-1) + 3(0)}{64 + 80 + 26 + 3} = -\frac{170}{173} \approx -0.9827 \tag{14}$$

Such a state-weighted value is phenomenologically indistinguishable from a static cosmological constant ($w = -1$) and entirely fails to capture the macroscopic degradation of the vacuum indicated by recent DESI observations.

However, weighting by volumetric state exclusion fundamentally mischaracterises the physical ontology of a discrete computational spacetime. In an active quantum error-correcting code, the computational work performed by the system to maintain the codespace (the physical vacuum) is not proportional to the dimensionality of the discarded Hilbert space. Rather, in accordance with Landauer's Principle [5], the algorithmic cost is strictly proportional to the number of active *syndrome measurements* (parity checks) that the lattice hardware must execute.

The four rules of the Holographic Circlette operate as four independent topological parity checks, evaluating distinct bit subspaces. At each discrete tick of the quantum walk, the lattice must expend an identical quantum of algorithmic work to evaluate the R4 parity check as it does to evaluate the R1 parity check, regardless of how many abstract states fail the condition.

Because algorithmic inertia and vacuum pressure are generated by the physical execution of these logic gates, the thermodynamic weight of each rule is exactly equal (1/4). Three of these syndrome evaluations successfully enforce unbroken topological symmetries, contributing perfectly to the negative pressure of the vacuum. Because the R4 syndrome measurement structurally leaks to permit neutrino mass generation via Feshbach resonance [6], it evaluates anomalously as pressureless topological dust ($w = 0$). Averaging the structural pressure over the four uniformly weighted operational parity checks yields the exact macroscopic limit:

$$w_0 = \frac{1}{4}(-1) + \frac{1}{4}(-1) + \frac{1}{4}(-1) + \frac{1}{4}(0) = -\mathbf{\frac{3}{4}} \tag{15}$$

This confirms that the $-3/4$ limit is not a statistical approximation, but an exact, rigid thermodynamic consequence of modelling the vacuum as a 4-syndrome continuous error-correcting algorithm.

5 Conclusion

The dynamical evolution of Dark Energy is an inherent consequence of modeling spacetime as an emergent informational matrix. By equating the unblemished, pre-error-corrected lattice at $a = 0$ to the perfect Lorentz-invariant vacuum ($w = -1$), and acknowledging the computational degradation of this vacuum to $w_0 = -3/4$ today, the Holographic Circlette framework formally derives the exact CPL evolutionary parameter $w_a = -1/4$.

This geometric solution resolves the contemporary crisis in cosmology: it provides a structural reason for $w_0 > -1$, it derives the exact phenomenological signature $w_a < 0$ favoured by the DESI 2024 dataset, and it successfully bounds both the past and future expansion of the universe, preventing catastrophic asymptotic infinities by treating vacuum energy as a finite topological resource.

References

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