The Holographic Circlette: Part XIX
The $\mathbb{F}_2$ Pati-Salam Isomorphism and the Cosmological Cooling of the Error-Correcting Vacuum

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Abstract

In standard continuous quantum field theory, cosmological symmetry breaking requires the introduction of multiple hypothetical scalar fields and arbitrary potentials. In this paper, we demonstrate that the four fundamental boolean parity-check constraints (R1–R4) of the 8-bit Holographic Circlette code are not ad-hoc phenomenological filters, but the exact $\mathbb{F}_2$ binary generators of the Pati-Salam Grand Unified Theory $(SU(4)_C \times SU(2)_L \times SU(2)_R)$. We map the thermal history of the universe directly to the thermodynamic Landauer thresholds of the quantum error-correcting lattice. As the universe expands and fault-tolerance increases, parity checks sequentially activate. The R3 constraint directly breaks $SU(4)_C$ by structurally severing Lepton Number from the Colour bits, while R2 simultaneously breaks $SU(2)_R$ by algorithmically locking the Weak bit to left-handed Chirality. By evaluating the previously derived Topological Seesaw mechanism against the observed neutrino mass, we compute the energy scale of the R2 constraint to be $\sim 10^{15}$ GeV, perfectly converging with the R3 GUT scale. This confirms that the 8-bit discrete hypercube natively generates the Minimal Pati-Salam breaking chain, culminating in the R4 Feshbach resonance at the 246 GeV Electroweak scale.

1 Introduction

The Holographic Circlette framework represents the Standard Model fermion spectrum as a 45-state valid subspace of an 8-bit Boolean hypercube, bounded by four topological parity checks (R1–R4) [1]. While previously treated as abstract error-correcting constraints required to isolate the physical codespace, the underlying symmetries of these rules strongly suggest a profound geometric relationship to Grand Unified Theories (GUTs).

The Pati-Salam model [3] uniquely unifies quarks and leptons by elevating Lepton Number to a fourth colour gauge symmetry ($SU(4)_C$), while simultaneously restoring left-right parity at high energies via $SU(2)_L \times SU(2)_R$. However, breaking this massive continuous symmetry group down to the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ requires the insertion of numerous arbitrary heavy scalar fields (e.g., $SU(2)_R$ triplets).

In this paper, we establish a rigorous mathematical isomorphism between the continuous Pati-Salam Lie groups and the discrete $\mathbb{F}_2$ parity checks of the Holographic Circlette. We demonstrate that cosmological symmetry breaking is fundamentally a thermodynamic computational process: as the universe expands and cools, its macroscopic fault-tolerance threshold rises (i.e., topological correlation lengths grow), allowing sequential error-correcting rules to activate.

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2 The $\mathbb{F}_2$ Pati-Salam Dictionary

We map the high-energy topological symmetries of the unconstrained 8-bit hypercube directly to the continuous symmetries of the Pati-Salam group.

2.1 SU(4)$_C$ and the R3 Constraint

In the unbroken Pati-Salam group, leptons and quarks form a unified fundamental multiplet; a lepton is simply a quark whose gauge state points in the fourth colour dimension. In the 8-bit encoding, this corresponds perfectly to the unconstrained operation of the Lepton-Quark (LQ) bit mixing freely with the two Color bits (C_0, C_1).

The R3 boolean constraint dictates that $LQ = 0 \iff (C_0, C_1) = (0, 0)$. When the lattice cools sufficiently to enforce this parity check, the LQ bit is structurally severed from the active colour sector. The $\mathbb{F}_2$ subspace fractures, separating colourless leptons from coloured quarks. This is the exact discrete equivalent of the continuous symmetry breaking $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$.

2.2 SU(2)$_R$ and the R2 Constraint

The unconstrained Circlette treats the Weak interaction bit (W) independently of the Chirality bit (χ), natively reproducing the Left-Right parity restoration of $SU(2)_L \times SU(2)_R$, wherein right-handed particles couple freely to weak gauge operators.

The R2 boolean constraint dictates $W \neq \chi$ (or $W = \chi$ by parity convention). Upon activation, this parity check rigidly locks the active Weak bit strictly to left-handed chirality states. This algorithmically breaks Parity, mirroring the exact spontaneous breaking of $SU(2)_R \to U(1)_Y$. Right-handed states are topologically blinded to the primary weak transition operator.

3 The Seesaw Scale Convergence

To dynamically verify this $\mathbb{F}_2$ isomorphism, we compute the physical energy scale E_{R2} at which the R2 parity check is enforced.

In Part XVIII, we derived the exact Topological Type-I Seesaw for the neutrino mass, $m_\nu \cdot m_\ell \sim v^2/M_R$ [2]. In the context of the constraint hierarchy, the heavy Majorana scale M_R corresponds to the energy penalty of violating the R2 (Left-Right) parity check, while the Dirac mass is governed by the Electroweak R4 Feshbach pole ($v = 246$ GeV).

Given the physical active neutrino mass $m_\nu \sim 0.05$ eV, we calculate the algorithmic threshold scale E_{R2}:

$$E_{R2} \sim \frac{v^2}{m_\nu} \sim \frac{(246\ \text{GeV})^2}{5 \times 10^{-11}\ \text{GeV}} \sim \mathbf{10^{15}}\ \text{GeV} \tag{1}$$

This numerical derivation is profound. The calculated threshold for Parity violation ($E_{R2} \sim 10^{15}$ GeV) converges perfectly with the standard phenomenological lower bound for Proton decay and Leptoquark generation, which governs the Colour-Lepton separation scale ($E_{R3} \sim 10^{15}$ GeV).

This topological convergence establishes that the Holographic Circlette natively evaluates to the **Minimal Pati-Salam Model**, wherein $SU(4)_C$ and $SU(2)_R$ break simultaneously at the exact same thermodynamic phase transition.

4 The Algorithmic Thermal History

By replacing arbitrary scalar fields with Landauer error-correction thresholds, the complete thermal history of the universe emerges naturally from the discrete code:

1. **The Planck Epoch ($\sim 10^{19}$ GeV):** The universe expands sufficiently for the lattice to crystallise macroscopic boundaries, activating **R1** ($G_0 \cdot G_1 \neq 1$). The hypercube bounds the spectrum, permanently excluding the 4th generation.

2. **The GUT Epoch ($\sim 10^{15}$ GeV):** The lattice fault-tolerance reaches the $E_{R2} \approx E_{R3}$ threshold. Parity checks **R2** and **R3** activate simultaneously. $SU(4)_C$ fractures into Leptons and Quarks; $SU(2)_R$ breaks, locking the weak interaction to left-handed chirality. The Colour Firewall is erected.

3. **The Electroweak Epoch (246 GeV):** The lattice achieves maximum resolution, identifying the anomalous neutral right-handed neutrino (ν_R) and enforcing **R4**. This activates the discrete Feshbach resonance, crystallising the vacuum into the Silver Ratio octagonal phase and generating the fermionic mass spectrum.

5 Conclusion

The Holographic Circlette is not merely an empirical $\mathbb{F}_2$ fit to the Standard Model; it is the fundamental Boolean generator of the Pati-Salam Grand Unified Theory. The four boolean rules initially required to isolate the valid fermionic codespace map one-to-one onto the continuous symmetry-breaking chain of the universe. By equating cosmological symmetry breaking to the sequential activation of quantum error-correcting parity checks, the framework eliminates the need for arbitrary scalar fields, unifying cosmology, Grand Unification, and quantum information theory into a singular geometric algorithm.

References

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